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The Rapid Transit Frequency and Fleet Size Setting Problem with Maximal Profit

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Abstract Metro and other Rapid Transit Systems (RTS) operate in cities and metropolitan areas, and their planning presents specific characteristics when compared to other railway networks. Often, the tracks of RTS lines are not interconnected, so network design and line planning (except frequency setting) are a unique step in the planning process. Then, frequency setting and fleet size determination are a joint problem that, moreover, has to be solved repeatedly. In this paper a mathematical programming model for this problem with the maximization of the net profit as objective function is presented.

1 Introduction

The sequential railway planning process is based on the knowledge of travel patterns, and mainly consists of four subsequent stages: network design, line planning, timetabling, and vehicle and crew scheduling (Michaelis and Schöbel (2009)). Network design consists of choosing, possibly from an underlying network, the stops/stations and tracks connecting them. Line planning aims at selecting, for each line, the two terminal stations, the itinerary, and the frequency. Rapid transit systems have characteristics belonging to both railway networks and public transit (Caprara et al (2007), Desaulniers and Hickman (2007)). In public transit and, particularly in rapid transit, the first step in line planning is to determine the routes of the lines, which implies the construction of the infrastructure (stations and sections). The frequencies are chosen in the second step. These two problems can often be solved jointly but, since the

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demand is time-dependent and elastic, the second one also has to be solved for the different periods of the day, days of the week, seasons, or whenever the demand changes. Because these demand changes, it is sometimes necessary to modify the frequency and the composition of the trains. Frequency and fleet size settings are two intertwined problems, and joint resolution seems to be plausible.

The objective functions regarding line planning, applied in the academic literature, can be grouped into two classes: customer oriented and cost oriented (see, Schöbel (2012)). Within the first class, you find the number of direct travelers, the traveling and riding time, and the number of transfers. In the second one, there are fixed and variable costs. For operating companies, costs are not the only factor to take into account. Low cost operations could not be profitable. Moreover, in the last years, an increasing concern on sustainability in transport planning can be observed. Thus, the relationship between costs and incomes becomes relevant in decisions regarding transportation projects. One way of evaluating this relationship is the net profit, defined as the difference between the revenue and the total cost. Moreover, since revenue depends on ridership, maximizing profit usually contributes to increasing the ridership, which is one of the most popular indicators of efficiency. The demand is given by an origin-destination matrix, and we also assume that a competing mode is functioning on the same space. This is one of the points that distinguishes our work from most of the line planning papers. We do not assume demand is captive, but it depends on the time comparison with the competing mode. In other words, ridership depends on the level of quality of the service offered.

The rest of the paper is structured as follows. Section 2 formally describes our line planning problem, which is later modeled as a mathematical programming program in Section 2.2. Because the model proposed is non-linear, in Section 3 we deal with this model by solving a number of linear problems. A computational experience in Section 4 finishes this paper.

2 The problem

2.1 Data and notation

In this section, we present the input data needed to define the Simultaneous Frequency and Capacity Problem (SFCP).

- Given is a set of connected lines $\mathcal{L} = \{\ell_1, \dots, \ell_{|\mathcal{L}|}\}$ in the RTS. Let $N = \{i_1, \dots, i_n\}$ be the set of stations that constitute the lines in $\mathcal{L}$. In railway terminology, a line is characterized by two terminal stops, its itinerary and the train size. Other important aspects of each line ℓ are its length, denoted by len_ℓ and measured in length units, and its number of stations associated, denoted by n_ℓ. Thus, the itinerary of each line $\ell \in \mathcal{L}$ can be represented as $\{(i_1, i_2), (i_2, i_3), \dots, (i_{n_\ell - 1}, i_{n_\ell})\}$, where i_1, i_{n_ℓ} are the terminal stations of the line, and $\{i_1, i_2, i_3, \dots, i_{n_\ell}\}$ and $\{i_{n_\ell}, i_{n_\ell - 1}, \dots, i_1\}$ define the two maximal paths of this line in the network.

- Each couple of (directed) arcs (i_{j_1}, i_{j_2}) and (i_{j_2}, i_{j_1}) define an (undirected) edge $\{i_{j_1}, i_{j_2}\}$. Let A be the set of (directed) arcs, and let $E = \{\{i,j\} : i,j \in N,\ i < j, (i,j) \text{ or } (j,i) \in A\}$ be the set of edges defined from A.
- From these sets, we describe a RTS as the graph $((N, E), \mathcal{L})$.
- Let d_{ij} be the length of each arc $(i, j) \in A$. We assume $d_{ij} = d_{ji}$. The parameter d_{ij} can also represent the time needed to traverse arc (i, j), transforming distances in times by means of the parameter λ, which represents the average distance traveled by a train in a hour (commercial speed). We assume the same value of λ for all trains. We consider a parameter ν_ℓ representing the cycle time of line ℓ, measured as the time needed for a train of line ℓ to go from the initial station to the final station and returning back. Thus, $\nu_\ell = 2 \cdot len_\ell/\lambda$.
- Let uc_i be the time spent in changing platforms at station i.
- Let $W = \{w_1, \ldots, w_{|W|}\} \subseteq N \times N$ be a set of ordered origin-destination (OD) pairs, $w = (w_s, w_t)$. For each OD pair $w \in W$, g_w is the expected number of passengers per hour for an average day and u_w^{ALT} is the travel time associated to w using the alternative mode, respectively.
- The passenger fare, the passenger subsidy (price that the government pays to the operator company for each trip) and the total number of hours that a train is operating per year are denoted by η, τ and ρ, respectively.
- The cost of operating one locomotive is c_{loc}, and the cost of operating one carriage is c_{carr}, both per unit of length. The crew cost c_{crew} per train and year is also given.
- The purchase cost of one locomotive is I_{loc}, and one carriage is I_{carr}. We assume in the horizon of $\hat{\rho}$ years the purchase of trains is recovered. We consider a minimum number $y^{\min}$ of carriages for each train.
- The capacity of a carriage is given by parameter Θ, measured in number of passengers seating and standing.
- A finite set of possible headways $\mathcal{H}$ for lines of the Rapid Transit System (RTS) is given.

2.2 Mathematical model

Our problem can be modeled as a mathematical programming program using the following sets of variables:

- $x_\ell \in \mathcal{H}$ is an integer variable representing the headway of line ℓ (time between services, expressed in minutes).
- $y_\ell \in \mathbb{Z}^+$ is the number of carriages used by each train of line ℓ.
- $u_w^{RTS} > 0$ represents the travel time of pair w using the RTS network.
- $p_w^{RTS} \in [0, 1]$ is the proportion of passengers of OD pair w using the RTS network, which depends on the travel time using the RTS (variable u_w^{RTS}) and on the travel time using the alternative mode (parameter u_w^{ALT}).
- $f_{ij}^{w\ell} = 1$ if the OD pair w traverses arc $(i, j) \in A$ using line ℓ, 0 otherwise. Note that these variables are set to zero whenever $(i, j) \notin \ell$, to reduce the size of the problem.

- $t_k^{w\ell\ell'} = 1$ if demand of pair w transfers at station k from line ℓ to line ℓ', 0 otherwise. Note that these variables are set to zero whenever k does not belong to the two lines, nor when k is the origin or destination of pair w, in order to reduce the size of the problem.
- B_ℓ is the required fleet of line ℓ.

The objective is the maximization of the net profit z_{NET}, defined as:

$$\begin{aligned}\text{Maximize}\Big[&\rho\hat{\rho}(\eta+\tau)\sum_{w\in W} g_w p_w^{RTS} \\ &-\rho\hat{\rho}\sum_{\ell\in\mathcal{L}}\lambda B_\ell(c_{loc}+y_\ell\cdot c_{carr}) \\ &-\sum_{\ell\in\mathcal{L}} B_\ell(I_{loc}+I_{carr}\cdot y_\ell) \\ &-\hat{\rho}\,c_{crew}\sum_{\ell\in\mathcal{L}} B_\ell\Big]\end{aligned} \tag{1}$$

The first term in (1) is the revenue z_{REV}, which depends on the number of passengers traveling (and therefore paying a ticket) in the RTS. The second term computes the rolling stock cost: the cost of operating the trains, which depends on the number of carriages. The last two terms are the fleet acquisition cost and the crew operating cost, respectively.

The constraints of the problem are:

$$t_k^{w\ell\ell'} \geq \sum_{j:(k,j)\in\ell} f_{kj}^{w\ell} + \sum_{i:(i,k)\in\ell'} f_{ik}^{w\ell'} - 1,\ w\in W, \ell\neq\ell'\in\mathcal{L}, k\in\ell\cap\ell',\ k\neq w_s, w_t \tag{2}$$

$$\sum_{\ell\in\mathcal{L}}\sum_{i:(i,k)\in\ell} f_{ik}^{w\ell} - \sum_{\ell\in\mathcal{L}}\sum_{j:(k,j)\in\ell} f_{kj}^{w\ell} = \begin{cases} 0,\ k\in N\setminus\{w_s,w_t\} \\ -1,\ k=w_s \\ +1,\ k=w_t \end{cases} \tag{3}$$

$$x_\ell \sum_{w\in W} g_w p_w^{RTS} f_{ij}^{w\ell} \leq 60\cdot\Theta\cdot y_\ell,\ \ \ell\in\mathcal{L}, \{i,j\}\in E \tag{4}$$

$$p_w^{RTS} = \frac{1}{1+e^{(\alpha-\beta(u_w^{ALT}-u_w^{RTS}))}},\ w\in W \tag{5}$$

$$u_w^{RTS} = \sum_{\ell\in\mathcal{L}}\sum_{j:\{w_s,j\}\in\ell} \frac{x_\ell f_{w_s j}^{w\ell}}{2} + (60/\lambda)\sum_{\ell\in\mathcal{L}}\sum_{\{i,j\}\in\ell} f_{ij}^{w\ell} d_{ij}$$
$$+\sum_{\ell\in\mathcal{L}}\sum_{\ell':\ell'\neq\ell}\sum_{i\in\ell\cap\ell'} t_i^{w\ell\ell'}(\frac{x_{\ell'}}{2}+uc_i),\ w=(w_s,w_t)\in W \tag{6}$$

$$B_\ell = \lceil 120\cdot len_\ell/x_\ell\lambda\rceil,\ \ell\in\mathcal{L} \tag{7}$$

$$y_\ell \geq y^{min},\ \ell\in\mathcal{L} \tag{8}$$

$$u_w^{RTS} > 0,\ w\in W$$
$$x_\ell\in\mathcal{H},\ \ell\in\mathcal{L}$$
$$f_{ij}^{w\ell}, t_k^{w\ell\ell'}\in\{0,1\}$$
$$k\in N, \{i,j\}\in E, (i,j)\in A, i\in N, \ell\in\mathcal{L}, w\in W.$$

Constraints (2) ensure that if an OD pair w enters station $k\in N$ using one line, and exits from this station using another line, then a transfer is done. Constraints (3) are the flow conservation constraints. Constraints (4) impose an upper bound on the maximum number of passengers that each line can transport per hour, which depends on the number of carriages and headway of this line. Constraints (5) represent the modal split, which uses the travel time described in Equation (6). Constraints (7) establish the required fleets as functions of the headways.

A lower bound on the number of carriages for each line is forced by Constraints (8).

The maximization of (1), subject to constraints (2)– (8), is a Mixed Integer Non-Linear Programming (MINLP) program that solves our problem. The non-linearities of this model will be specified, as well as some ways to avoid them.

3 Algorithm

In this section we present an algorithm for solving the problem described in Section 2, based on efficient approaches of the mathematical model.

As mentioned before, the MINLP presents several nonlinearities. In the following, we describe such nonlinearities as well as the way in which we dealt with them, so as to describe them as linear constraints.

1. In Constraints (4), the binary variable $f_{ij}^{w\ell}$ is multiplying the positive variable p_w^{RTS}. This product can be easily linearized, by defining a new set of variables $q_{ij}^{w\ell}$ as follows:

$$q_{ij}^{w\ell} \leq f_{ij}^{w\ell}, \ell \in \mathcal{L}, \{i,j\} \in \ell, w \in W \tag{9}$$

$$p_w^{RTS} - (1 - f_{ij}^{w\ell}) \leq q_{ij}^{w\ell},\ \ell \in \mathcal{L}, \{i,j\} \in \ell, w \in W \tag{10}$$

$$q_{ij}^{w\ell} \leq p_w^{RTS},\ \ell \in \mathcal{L}, \{i,j\} \in \ell, w \in W. \tag{11}$$

2. The definition of the proportion of passengers using the RTS, Constraints (5), uses the non-linear function *logit.* This nonlinearity is avoided by approximating the logit function by a linear function which takes into account three intervals on its abscissa axis as follows. Let z be the variable u_w^{RTS} representing the travel time in the RTS and let $F(z) = 1/(1 + \exp(\alpha - \beta(u_w^{ALT} - z))$ be the corresponding logit function for z. The piecewise linear function is defined as

$$\mathcal{P}(z) := \begin{cases} 1, & z < u_w^{ALT} - 2/\beta \\ -\beta/4z + (2 + \beta u_w^{ALT})/4, & z \in [u_w^{ALT} - 2/\beta, u_w^{ALT} + 2/\beta] \\ 0, & z \geq u_w^{ALT} + 2/\beta. \end{cases}$$

3. The required fleet described in Constraints (7), uses the ceiling function, which is non-linear as well, and the headway variables are in the denominator. This last nonlinearity can be avoided by fixing the headway of each line as a parameter.

Let $ILP(x_1, ..., x_{|\mathcal{L}|})$ be the model obtained after avoiding the two first nonlinearities, in which the headway of each line ℓ, $x_\ell \in \mathcal{H}$ is fixed as a parameter. The reader may note that the resulting program is an Integer Linear Programming model.

Then, the algorithm presented in this section solves $ILP(x_1, ..., x_{|\mathcal{L}|})$, for all feasible combinations of headways $(x_1, ..., x_{|\mathcal{L}|}) \in \mathcal{H}^{|\mathcal{L}|}$, keeping as a final output the best solution found. The solution procedure is shown in Algorithm 1.

4 Computational experiments

In this section we show a computational experience, conducted over three different topologies, called 6×2, 7×3, and 8×3, where $n \times m$ means a

Data: Input for our problem
for *each combination of headways* $(x_1, ..., x_{|\mathcal{L}|})$ **do**
    solve $ILP(x_1, ..., x_{|\mathcal{L}|})$;
end
Result: $\arg \max_{(x_1,...,x_{|\mathcal{L}|})} ILP(x_1, ..., x_{|\mathcal{L}|})$.

Algorithm 1: Pseudocode for the ILP-based algorithm.

network with n nodes and m lines. These topologies are described in figures 1, 2, and 3.

Ten instances were randomly generated for each configuration, having in total 30 instances to solve, as follows. The number of passengers of each OD pair w was obtained following a uniform distribution. To define each arc length, the coordinates of each station were set randomly by means of another uniform distribution. So, the arc length at each instance is different since each arc connects to different positions of stations. The travel times u_w^{ALT} using the alternative mode, were obtained by means of the Euclidean distance and a speed of 20 km/h, whereas, the travel time in the RTS were obtained according to the riding times with a speed of 30 km/h, the waiting time and the transfer time. Costs (both purchase and operation) are based on the specific train model *Civia* as in De-Los-Santos et al (2014). The parameters of the logit function were set to $\alpha = -0.3$ and $\beta = 1$, like in Marín and García-Ródenas (2009). Table 1 summarizes the input data for our experiments.

All the calculations for Algorithm 1 were performed in GAMS/CPLEX, in a computer with 8 Gb of RAM memory and 2.8 Ghz CPU.

The reader may note that these results heavily depend on the input data. Our experiments aim at validating the algorithm. A real application (using real data) will help us better understand its functioning. We also noted that the variations in net profit and revenue are not large, when changing the configuration. It is also interesting to note how the computational times increase by one order of magnitude (roughly speaking) from the configuration 6x2 to the configuration 7x3. Such increase is not as high when going from 7x3 to 8x3. A more detailed analysis of these time increments is left for further research.

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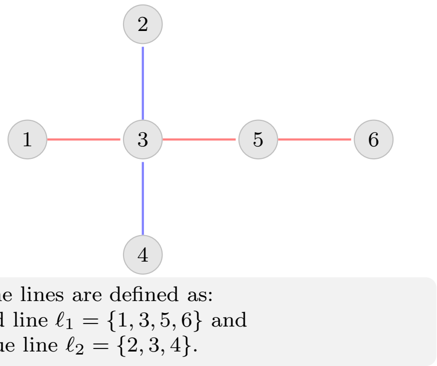

Fig. 1 Representation of 6×2-configuration.

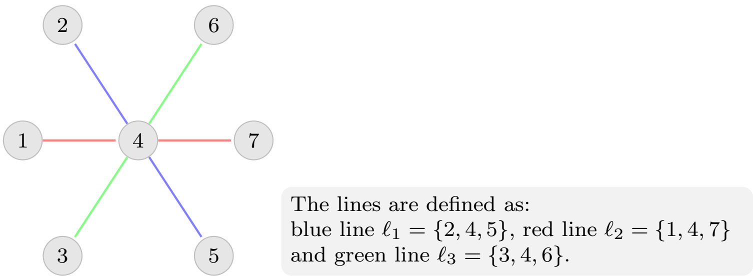

Fig. 2 Representation of 7×3-configuration.

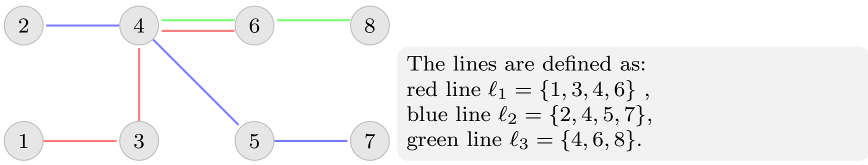

Fig. 3 Representation of 8×3-configuration.

Parameters		
Name	**Description**	**Value**
$\hat{\rho}$	years to recover the purchase	20
ρ	number of operative hours per year	6935
c_{loc}	costs for operating one locomotive per kilometer [€/km]	34
c_{carr}	operating cost of a carriage per kilometer [€/km]	2
c_{crew}	per crew and year for each train [€/ year]	$75 \cdot 10^3$
I_{loc}	purchase cost of one locomotive in €	$2.5 \cdot 10^6$
I_{carr}	purchase cost of one carriage in €	$0.9 \cdot 10^6$
Θ	capacity of each carriage (number of passengers)	$2 \cdot 10^2$
x_ℓ	possible values	{5,10,15,20}

Table 1 Model parameters for the experiment.

instance	z_{NET}	z_{REV}	x_ℓ	y_ℓ	CPU time	trips
$6x2_1$	3.20E+09	4.88E+09	[5,5]	[3,2]	2.04	10049
$6x2_2$	3.69E+09	5.37E+09	[5,5]	[3,2]	2	11054
$6x2_3$	4.80E+09	6.48E+09	[5,5]	[3,2]	2.43	13341
$6x2_4$	4.43E+09	6.11E+09	[5,5]	[3,2]	2	12591
$6x2_5$	3.04E+09	4.68E+09	[5,5]	[3,1]	2	9649
$6x2_6$	5.08E+09	6.87E+09	[5,5]	[5,2]	2,04	14148
$6x2_7$	4.42E+09	6.15E+09	[5,5]	[4,2]	2.82	12672
$6x2_8$	4.27E+09	6.00E+09	[5,5]	[4,2]	2.49	12357
$6x2_9$	3.96E+09	5.64E+09	[5,5]	[3,2]	2.03	11608
$6x2_{10}$	5.65E+09	7.38E+09	[5,5]	[4,2]	2.08	15195
$7x3_1$	4.00E+09	7.32E+09	[5,5,5]	[3,2,2]	12.84	15075
$7x3_2$	2.96E+09	6.21E+09	[5,5,5]	[2,2,2]	11.12	12783
$7x3_3$	3.55E+09	6.53E+09	[5,5,5]	[3,2,2]	15.16	13447
$7x3_4$	2.65E+09	5.90E+09	[5,5,5]	[2,2,2]	11	12151
$7x3_5$	3.75E+09	6.75E+09	[5,5,5]	[3,2,2]	15.48	13904
$7x3_6$	4.51E+09	7.49E+09	[5,5,5]	[3,2,2]	11.29	15423
$7x3_7$	2.72E+09	5.99E+09	[5,5,5]	[3,1,2]	10.88	12334
$7x3_8$	5.07E+09	8.39E+09	[5,5,5]	[3,2,2]	11.2	17280
$7x3_9$	3.37E+09	6.30E+09	[5,5,5]	[2,2,2]	14.97	12976
$7x3_{10}$	4.82E+09	8.15E+09	[5,5,5]	[3,2,2]	15.04	16779
$8x3_1$	2.88E+09	4.83E+09	[15,5,5]	[1,3,1]	17.69	9946
$8x3_2$	2.66E+09	4.95E+09	[15,5,5]	[1,3,1]	19.01	10198
$8x3_3$	2.33E+09	4.31E+09	[15,5,5]	[1,3,2]	18.37	8881
$8x3_4$	2.73E+09	4.71E+09	[15,5,5]	[1,3,2]	18.5	9709
$8x3_5$	2.35E+09	4.91E+09	[5,5,5]	[1,3,1]	17.49	10118
$8x3_6$	3.04E+09	5.60E+09	[5,5,5]	[1,3,1]	17.15	11544
$8x3_7$	3,11E+09	5.71E+09	[5,5,5]	[1,3,2]	16.95	11753
$8x3_8$	3.17E+09	5.16E+09	[15,5,5]	[1,3,2]	17.55	10623
$8x3_9$	3.22E+09	5.84E+09	[5,5,5]	[2,3,1]	16.72	12031
$8x3_{10}$	3.53E+09	6.10E+09	[5,5,5]	[1,3,1]	18.49	12561

Table 2 Summary of results for the mathematical model.