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Timetabling and Passenger Routing in Public Transport

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Abstract The task of timetabling is to schedule the trips in a public transport system by determining periodic arrival and departure times at every station. The goal is to provide a service that is both attractive for passengers and can be operated economically. To date, timetable optimization is generally done with respect to fixed passenger routes, i.e., it is assumed that passengers do not respond to changes in the timetable. This is unrealistic and ignores potentially valuable degrees of freedom. We investigate in this paper periodic timetabling models with integrated passenger routing. We propose several models that differ in the allowed passenger paths and the objectives. We compare these models theoretically and report on computations on real-world instances for the city of Wuppertal.

Keywords Passenger routing · Periodic timetabling · Public transport

1 Introduction

The strategic planning process in public transport is usually subdivided into consecutive planning steps of network design, line planning, and timetabling. In each of these planning steps there are two main objectives, namely, minimization of operation costs and minimization of passenger discomfort. The latter objective is usually measured in terms of quantities such as travel time, number of transfers, or transfer time, that depend on travel choices, whose forecast in turn requires a consideration of human behavior. This is clearly just as difficult as it is important. The integration of passenger behavior into network design, line planning, and timetabling models is therefore a major

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challenge in public transit optimization. First approaches have been made in the area of line planning: Integrated line planning and passenger routing models have been proposed by Schöbel and Scholl (2006), Borndörfer et al (2007), and Borndörfer and Karbstein (2012), the last reference reports also on successful computations.

Timetable optimization has mostly been studied with respect to a fixed passenger routing based on path lengths in the network, see, e. g., Liebchen (2006), Lindner (2000), and Nachtigall (1998). Passengers, however, usually choose their routes depending on the timetable. This topic has been taken up only recently. For aperiodic timetabling, Schmidt (2012) studies the complexity of integrating passenger routings. She develops an exact solution approach for the case where the first and last train of each passenger path are fixed, see also Schmidt and Schöbel (2014). The only approaches to integrated passenger routing and periodic timetabling that we are aware of are the Master theses of Kinder (2008), Lübbe (2009), and Siebert (2011). Kinder investigates a heuristic approach that is based on a time-expanded event-activity network. Iteratively computing timetables and rerouting the passengers, the method converges towards a local optimum. Lübbe proposes an integrated quadratic model and linearizes it to obtain an integer linear programming model. His computations indicate a potential for travel time improvements but he could only deal with very small instances. Siebert provides worst case error analyses and compares an integrated integer programming model with an iterated approach.

The aim of this paper is to investigate the impact of routing decisions on timetable optimization in analogy to the work of Pfetsch and Borndörfer (2006) for the line planning case. To this purpose, we propose an integer programming approach to the integrated timetabling and passenger routing problem. We compare the differences between arbitrary passenger routings and passenger routings on shortest paths w.r.t. the network. Like most passenger-oriented models, these approaches minimize the total travel time for all passengers in the sense of a system optimum. This can lead to timetables in which the average travel time for all passengers is small, while some passengers are heavily disadvantaged. We therefore propose to consider also the maximum travel time. We test our models on real-world instances for the city of Wuppertal.

2 Periodic Timetabling with Fixed Passenger Routing

Most models in the literature for the periodic timetable problem are based on the *periodic event scheduling problem* (PESP) developed by Serafini and Ukovich (1989). We consider the following extended version. We are given a directed graph $N = (V, A)$, the *event-activity network*. The nodes V are called *events* and represent arrivals and departures of lines at their stations, i.e., $V = V_{\text{arr}} \cup V_{\text{dep}}$. The arcs $A \subseteq V \times V$ are called *activities* and model driving between stations, waiting at stations, and possible transfers between lines at stations, i.e., $A = A_{\text{drive}} \cup A_{\text{dwell}} \cup A_{\text{trans}}$. Further, we are given lower and upper time bounds $\ell_a, u_a \in \mathbb{Q}_{\geq 0}$, respectively, for the duration of activity $a \in A$. Passengers can start and end their trips in V_{dep} and V_{arr}, respectively. The passenger demand is given in terms of an *origin-destination matrix* (OD-matrix) $(d_{st}) \in \mathbb{Q}_{\geq 0}$ specifying for each pair $(s,t) \in V_{\text{dep}} \times V_{\text{arr}}$ the number of passengers that want to travel from s to t. Let $\mathcal{D} = \{(s,t) \in V_{\text{dep}} \times V_{\text{arr}} : d_{st} > 0\}$ be the set of all *OD-pairs* and for an OD-pair (s,t) let $\mathcal{P}_{st}$ be the set of (s,t)-paths in N and $\mathcal{P} := \bigcup_{(s,t)\in\mathcal{D}} \mathcal{P}_{st}$ be the set of all passenger paths.

A *periodic timetable* $\pi : V \to \mathbb{R}$ determines arrival and departure times at all arrival and departure nodes, respectively, that are assumed to repeat periodically w.r.t. to a *period time* $T \in \mathbb{R}_{\geq 0}$. Given $x \in \mathbb{R}$, we define the modulo operator by $[x]_T := \min\{x + zT : x + zT \geq 0, z \in \mathbb{Z}\}$. We call a timetable *feasible* if the *periodic interval constraints*

$$[\pi_w - \pi_v - \ell_a]_T \in [0, u_a - \ell_a] \quad \forall\, a = (v,w) \in A$$

are satisfied. We assume w.l.o.g. that $\ell_a < T$ and $u_a - \ell_a < T$ for all $a \in A$.

Let $\mathcal{P}'_{st} \subseteq \mathcal{P}_{st}$ and $\mathcal{P}' := \bigcup_{(s,t)\in\mathcal{D}} \mathcal{P}'_{st} \subseteq \mathcal{P}$ be subsets of passenger paths that model routing restrictions. For a feasible timetable π, the time duration of activity $a \in A$ is given by $x_a := \ell_a + [\pi_w - \pi_v - \ell_a]_T$, and the time duration or *travel time* of a passenger path $p \in \mathcal{P}'$ is $x_p := \sum_{a\in p} x_a$. If y_p passengers travel on path $p \in \mathcal{P}'$, the total travel time of all passengers is $\sum_{p\in\mathcal{P}'} x_p y_p$. The goal is to find a feasible timetable such that the total travel time, assuming passengers travel on shortest paths in P', is minimized.

Introducing timetable variables π_v for the timing of event v, x_a for the duration of activity a, and passenger variables y_p for the number of passengers that travel on path $p \in \mathcal{P}$, we can state the following mixed-integer non-linear program with congruence relations for the integrated passenger routing and timetabling problem:

$$\text{(PTT)} \quad \min \sum_{(s,t)\in\mathcal{D}} \sum_{p\in\mathcal{P}'_{st}} \sum_{a\in p} d_{st}\, x_a\, y_p$$

$$\text{s.t.} \quad [\pi_w - \pi_v - \ell_a]_T \leq u_a - \ell_a \quad \forall\, a = (v,w) \in A \quad (1)$$

$$[\pi_w - \pi_v - \ell_a]_T + \ell_a = x_a \quad \forall\, a = (v,w) \in A \quad (2)$$

$$\sum_{p\in\mathcal{P}'_{st}} y_p = 1 \quad \forall\, (s,t) \in \mathcal{D} \quad (3)$$

$$\pi_v \geq 0 \quad \forall\, v \in V \quad (4)$$

$$y_p \geq 0 \quad \forall\, p \in \mathcal{P}'. \quad (5)$$

The model (PTT) minimizes the total passenger travel time. Constraints (1) guarantee a feasible timetable. Constraints (3) enforce the passenger flow.

We remark that conditions (1) and (2) can be formulated in terms of linear constraints, using additional integer periodic offset variables for each activity, see, e.g., Liebchen (2006). An alternative linearization, which we use for our computations in Section 5, is obtained by transforming the event-activity network into a time-expanded event-activity network, see, e.g., Kinder (2008).

2.1 Timetabling Models

We derive variants of (PTT) by specifying the set of passenger paths and including capacity constraints.

We obtain a *shortest path routing model* (SPR) by setting $\mathcal{P}' := \mathcal{P}$, i.e., the passengers travel along the shortest path w.r.t. travel times induced by the timetable. In the *lower-bound routing model* (LBR), on the other hand, $\mathcal{P}'$ contains for each $(s,t) \in \mathcal{D}$ only the shortest path w.r.t. the lower bounds of the activities. That is $\mathcal{P}'_{st} := \arg\min \left\{ \sum_{a \in p} \ell_a : p \in \mathcal{P}_{st} \right\}$.

To derive capacitated versions of these models, we include a capacity $\kappa_a \in \mathbb{Q}_{\geq 0}$ for each activity $a \in A$ and require that the passenger flow does not exceed it by adding the following constraints:

$$\sum_{(s,t) \in \mathcal{D}} \sum_{p \in \mathcal{P}'_{st} : a \in p} d_{st}\, y_p \leq \kappa_a \quad \forall\, a \in A. \tag{6}$$

The *capacitated multi-path routing model* (κ-MPR) is obtained by setting $\mathcal{P}' := \mathcal{P}$ and including the capacity constraints (6). For the *capacitated unsplittable path routing model* (κ-UPR) we also set $\mathcal{P}' := \mathcal{P}$ and include the capacity constraints (6). Additionally, we require $y_p \in \{0,1\}$ for all $p \in \mathcal{P}$, that is, all passengers corresponding to an OD-pair $(s,t) \in \mathcal{D}$ have to travel on the same (s,t)-path.

3 Minimizing the Total Travel Time

In this section, we investigate the influence of routing restrictions on the travel time minimum and, later, study the impact of an alternative objective.

We use the following further notation. Denote by $v(M; I)$ the optimal objective value of a model $M \in \{\text{SPR}, \text{LBR}, \kappa\text{-MPR}, \kappa\text{-UPR}\}$ and an instance I. We denote by

$$\text{gap}(M_1, M_2) := \sup_I \frac{v(M_1; I)}{v(M_2; I)}$$

the *gap* between the optimal objective values of the models M_1 and M_2, where the supremum is taken over all instances I. The definitions of the models imply immediately for any instance I:

$$\begin{aligned} \text{gap}(\text{LBR}, \text{SPR}) \geq 1 \qquad &\Leftrightarrow \qquad v(\text{SPR}; I) \leq v(\text{LBR}; I) \\ \text{gap}(\kappa\text{-UPR}, \kappa\text{-MPR}) \geq 1 \qquad &\Leftrightarrow \qquad v(\kappa\text{-MPR}; I) \leq v(\kappa\text{-UPR}; I). \end{aligned} \tag{7}$$

We show in the following that there are instances such that the inequalities (7) are strict and that, indeed, the gap can be arbitrarily large.

Theorem 1 $\text{gap}(\text{LBR}, \text{SPR}) = \infty$.

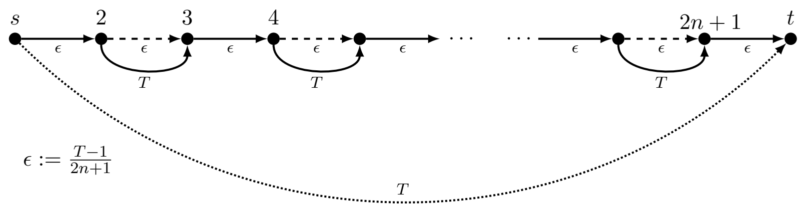

Fig. 1 Instance for Theorem 1.

Proof Consider the directed graph D in Figure 1. D has $2n + 2$ nodes and $2n + 1 + n + 1 = 3n + 2$ arcs, $n \in \mathbb{N}$. Based on D we construct a timetabling instance I by associating the nodes with stations and the arcs with driving activities of lines (to be defined in a minute); arcs corresponding to transfer and dwell activities are omitted in Figure 1.

We define activity times as follows. For all transfer activities, the lower time bound is zero and the upper time bound is $T \in \mathbb{N}$. The lower and the upper time bound of all line dwell activities at each station is zero. For each line driving activity the lower time bound equals the upper time bound. Hence, this timetable problem reduces to determining for each line the departure time at its first station and to routing the passengers.

We associate $n + 2$ lines with the arcs of D. There is one line from s to t (dotted arc) with a driving time of T and no intermediate stations. There is a second line (solid arcs) from s to t with $2n$ intermediate stations. The driving time between the stops of this line is alternatingly $\epsilon := \frac{T-1}{2n+1}$ and T. Between every two stations, for which the driving time of the second line is T, there is another line with a driving time of only ϵ (dashed arcs). There is only one passenger that wants to travel from s to t.

First consider model (LBR). In any solution of (LBR), the passenger is routed along the unique shortest (s, t)-path with respect to the driving time and transfer times of zero. This path uses all upper arcs with a driving time of ϵ and would have a total length of $(2n + 1)\epsilon = T - 1$, if the transfer times at all stations would be zero. However, there is no feasible timetable for this instance such that the transfer time at every station in this path is zero. In particular, in any solution of (LBR), the transfer times at stations 2 and 3 sum up to

$$T - \epsilon$$

as for every following pair of stations along this path. Hence, the travel time for this path is in total $T - 1 + n(T - \epsilon)$ and $v(\text{LBR}; I) = T - 1 + n(T - \epsilon)$. In an optimal solution to (SPR) the passenger travels on the bottom line with a travel time of T for any timetable and, hence, $v(\text{SPR}; I) = T$. We can conclude

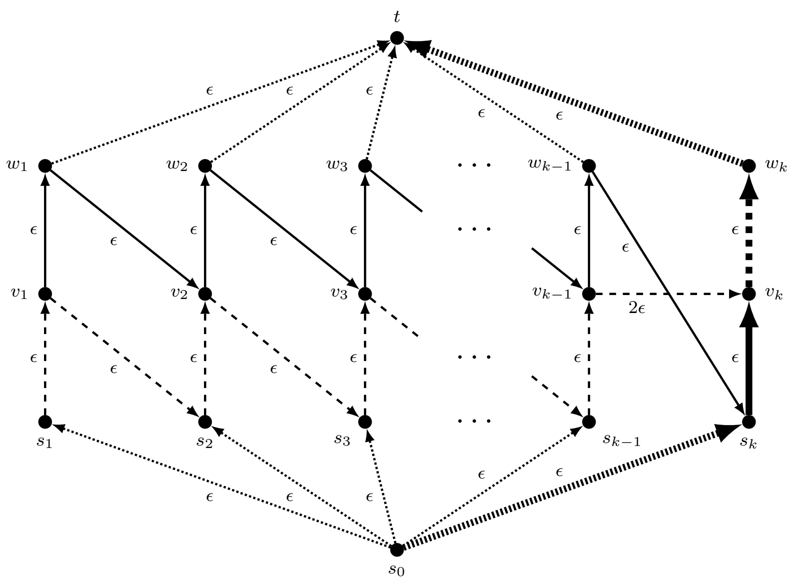

Fig. 2 Instance for Theorem 2. All arcs in this graph have a capacity of k.

that

$$\frac{v(\mathrm{LBR}; I)}{v(\mathrm{SPR}; I)} = \frac{T - 1 + n(T - \epsilon)}{T}$$

$$= n + \frac{(T-1)(1 - \frac{n}{2n+1})}{T} \quad \xrightarrow[n\to\infty]{} \quad \infty,$$

which proves the claim. □

Theorem 2 gap(κ-UPR, κ-MPR) $= \infty$.

Proof Consider the directed graph D in Figure 2. Similar to the proof of Theorem 1, we construct a timetabling instance I based on D. This instance contains $2k + 2$ lines, $k \in \mathbb{N}$. $2k$ lines are represented by the dotted arcs $\{(s_0, s_i)\}_{1\leq i\leq k}$ and $\{(w_i, t)\}_{1\leq i\leq k}$. Then there is one line (dashed arcs) starting in s_1 and ending in w_k and the last line (solid arcs) is from v_1 to v_k. Again, the time bounds for all dwell activities are zero, the lower bound for transfer activities is zero, and for all driving activities the lower time bound equals the upper time bound. In particular, the duration of all driving activities is $\epsilon > 0$ except for the activity corresponding to the arc (v_{k-1}, v_k) that has a duration of 2ϵ. All transfer and dwell activities have infinite capacity. All driving activities have a capacity of k. We set the passenger demand to $d_{s_i,t} = 1$ for each $1 \leq i \leq k-1$, and we set $d_{s_0,t} = k$; all other demands are set to zero.

First consider problem (κ-UPR). For any timetable, the passengers that want to go from s_i to t travel along paths that must start with the arc (s_i, v_i), for all $1 \leq i \leq k-1$. Then, these arcs have only $k-1$ capacity left and cannot be used any more by the k passengers that want to go from s_0 to t. These passengers have to travel via the path (s_0, s_k, v_k, w_k, t) since it is the only (s_0, t)-path with sufficient capacity that is left. These passengers block the arc (v_k, w_k), such that all passengers that want to go from s_i to t, $1 \leq i \leq k-1$, must transfer at some node v_i, $1 \leq i \leq k-1$ (different from v_k). The dashed and the solid line are constructed in such a way that the sum of the transfer times at nodes v_{k-1} and v_k is at least $T - \epsilon$. Moreover, the transfer times at nodes v_i, $1 \leq i \leq k-1$, are all identical. Hence, there is a minimum total transfer time of all passengers of at least $(k-1)(T-\epsilon)$, while the minimum total driving time is at least $(k-1)3\epsilon + k \cdot 4\epsilon$. If the passengers from s_i to t travel along the paths (s_i, v_i, w_i, t), these values can indeed be achieved by synchronizing the dashed and the dotted lines at node v_k, namely, the solid line can depart at v_1 at time 0 and the dashed line can depart at s_1 also at 0. Hence, the minimum total travel time (achieved for this timetable) is

$$v(\kappa\text{-UPR}; I) = (k-1)3\epsilon + k \cdot 4\epsilon + (k-1)(T-\epsilon) = 6k\epsilon - 2\epsilon + kT - T.$$

In an optimal solution to (κ-MPR), the passengers from s_0 to t can split and travel along $k-1$ paths via v_i, $1 \leq i \leq k-1$. The transfer time in an optimal timetable for these passenger paths at v_i can be zero for all $1 \leq i \leq k$ (the solid line can depart at v_1 at time ϵ and the dashed line can depart at s_1 at time 0). The minimum total travel time for all passengers is therefore

$$v(\kappa\text{-MPR}; I) = (k-1)3\epsilon + k \cdot 4\epsilon = 7k\epsilon - 3\epsilon.$$

We set $\epsilon := \frac{1}{k}$ and can conclude that

$$\frac{v(\kappa\text{-UPR}; I)}{v(\kappa\text{-MPR}; I)} = \frac{6k\epsilon - 2\epsilon + kT - T}{7k\epsilon - 3\epsilon} = \frac{6 - \frac{2}{k} + kT - T}{7 - \frac{3}{k}} \quad \underset{k\to\infty}{\longrightarrow} \quad \infty.$$

This finishes the proof. □

4 Minimizing the Maximum Travel Time

In this section we consider an alternative objective function, namely to minimize the maximum travel time among all passengers. To this purpose, we introduce an additional model variant. The *min-max travel time shortest path routing model* (SPR$^{\max}$) is obtained from SPR by adding a variable $\tau^{\max} \in \mathbb{R}$, representing the maximum weighted travel time among all OD-pairs, and a corresponding constraint

$$\sum_{p \in \mathcal{P}'_{st}} \sum_{a \in p} d_{st}\, x_a\, y_p \leq \tau^{\max} \quad \forall\, (s,t) \in \mathcal{D}$$

and by changing the objective function to

$$\min\ \tau^{\max}.$$

For a problem $M \in \{\text{SPR}, \text{SPR}^{\max}\}$ and an instance I let $\text{opt}(M; I)$ be the set of time duration variables x and passenger variables y that give rise to an optimal solution. Then we denote by

$$\tau^{\max}(M; I) := \max \left\{ \sum_{p \in \mathcal{P}_{st}} \sum_{a \in p} d_{st}\, x_a^*\, y_p^* : (s,t) \in \mathcal{D}, (x^*, y^*) \in \text{opt}(M; I) \right\}$$

the maximum weighted travel time among all OD-pairs in any optimal solution and by

$$\tau^{\text{total}}(M; I) := \max \left\{ \sum_{(s,t) \in \mathcal{D}} \sum_{p \in \mathcal{P}_{st}} \sum_{a \in p} d_{st}\, x_a^*\, y_p^* : (x^*, y^*) \in \text{opt}(M; I) \right\}$$

the maximum total weighted travel time of all passengers induced by the timetable in any optimal solution to M for instance I. Note that by definition

$$\begin{aligned} v(\text{SPR}^{\max}; I) &= \tau^{\max}(\text{SPR}^{\max}; I) \\ v(\text{SPR}; I) &= \tau^{\text{total}}(\text{SPR}; I) \\ \tau^{\max}(\text{SPR}^{\max}; I) &\leq \tau^{\max}(\text{SPR}; I) \\ \tau^{\text{total}}(\text{SPR}; I) &\leq \tau^{\text{total}}(\text{SPR}^{\max}; I) \end{aligned}$$

holds for every instance I. We denote by

$$\text{gap}^{\max}(\text{SPR}, \text{SPR}^{\max}) := \sup_I \frac{\tau^{\max}(\text{SPR}; I)}{\tau^{\max}(\text{SPR}^{\max}; I)}$$

the *gap* between the maximum weighted travel time among all OD-pairs in any optimal solution of the models SPR and $\text{SPR}^{\max}$, respectively, and by

$$\text{gap}^{\text{total}}(\text{SPR}^{\max}, \text{SPR}) := \sup_I \frac{\tau^{\text{total}}(\text{SPR}^{\max}; I)}{\tau^{\text{total}}(\text{SPR}; I)},$$

the *gap* between the maximum total weighted travel travel time in any optimal solution of the models $\text{SPR}^{\max}$ and SPR, respectively. The supremum is taken over all instances I.

Theorem 3 $\text{gap}^{\max}(\text{SPR}, \text{SPR}^{\max}) = \infty$.

Proof Consider the directed graph D in Figure 3. D has $3k$ nodes and $4k - 2$ arcs, $k \in \mathbb{N}$. Based on D we construct a timetabling instance I as follows. Again, we set the upper and lower bounds on the duration for all activities such that this timetabling problem reduces to determining for each line the departure time at its first station and to routing the passengers (fixed durations for all driving activities, zero duration for all dwell activities). We associate two lines with the arcs of D. There is one line (dashed arcs) from s_1 to v_k with a driving time of $\epsilon = \frac{T+1}{k}$, $T \in \mathbb{N}$, on each driving activity. The second line (solid arcs) starts in v_1 and ends in t_k. The duration for each driving activity

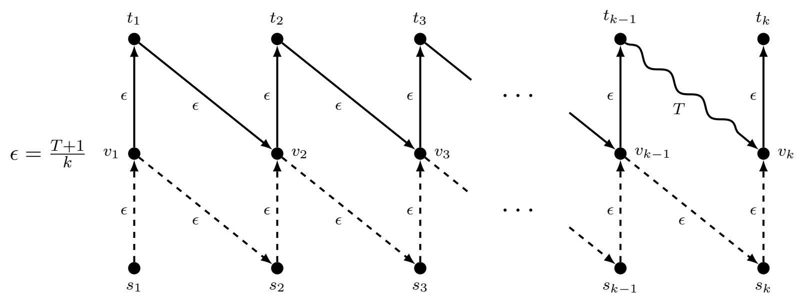

Fig. 3 Instance for Theorem 3.

of this line equals ϵ except for the second last arc from t_{k-1} to v_k that has a driving time of T. We set the passenger demand for each OD-pair (s_i, t_i), $1 \leq i \leq k$, to one; all other demands are set to zero.

For each OD-pair $(s_i, t_i) \in \mathcal{D}$, $1 \leq i \leq k$, there exists only a single path from s_i to t_i via the node v_i. Hence, the driving time for each OD-pair is 2ϵ for any timetable. The dashed and the solid line are constructed in such a way that the transfer times at nodes v_i, $1 \leq i \leq k-1$, are all identical. Moreover, if the two lines are synchronized at node v_k, then the transfer times at nodes v_i, $1 \leq i \leq k-1$, are all equal to ϵ. This would yield a total transfer time of $(k-1)\epsilon = T - \frac{T+1}{k} + 1$. If a timetable synchronizes the lines at the nodes v_i, $1 \leq i \leq k-1$, on the other hand, the transfer time at node v_k is $T - \epsilon = T - \frac{T+1}{k}$.

First consider problem (SPR). In an optimal solution, the departure time of the dashed line in s_1 is 0 and the solid line departs in v_1 at ϵ, such that the two lines are synchronized at the nodes v_i, $1 \leq i \leq k-1$. The resulting transfer time for the pair (s_k, t_k) at v_k equals $T - \epsilon$. Hence, this OD-pair yields the maximum travel time of $T + \epsilon$ among all OD-pairs for this timetable.

In an optimal solution to problem (SPR$^{\max}$), the lines are synchronized at node v_k by setting the departure time of the dashed line at s_1 to 0 and the departure time of the solid line at v_1 to $2\,\epsilon$. The resulting transfer time for each OD-pair (s_i, t_i) at v_i with $1 \leq i \leq k-1$ is ϵ and for the pair (s_k, t_k) the transfer time at v_k is zero. The travel time for all OD-pairs (s_i, t_i) with $1 \leq i \leq k-1$ is $3\,\epsilon$, which gives the maximum travel time. We can conclude that

$$\frac{\tau^{\max}(\text{SPR}; I)}{\tau^{\max}(\text{SPR}^{\max}; I)} = \frac{T+\epsilon}{3\epsilon} = \frac{T + \frac{T+1}{k}}{3\frac{T+1}{k}} = \frac{(k+1)T+1}{3T} \xrightarrow[k\to\infty]{} \infty,$$

which proves the claim. □

Note that the total travel time of the (SPR) solution is $\tau^{\text{total}}(\text{SPR}; I) = (k-1)2\epsilon + T - \epsilon = 3T + 2 - 3\frac{T+1}{k}$ and the total travel time of the (SPR$^{\max}$) solution is $\tau^{\text{total}}(\text{SPR}^{\max}; I) = (k-1)3\epsilon + 2\epsilon = 3T + 3 - \frac{T+1}{k}$.

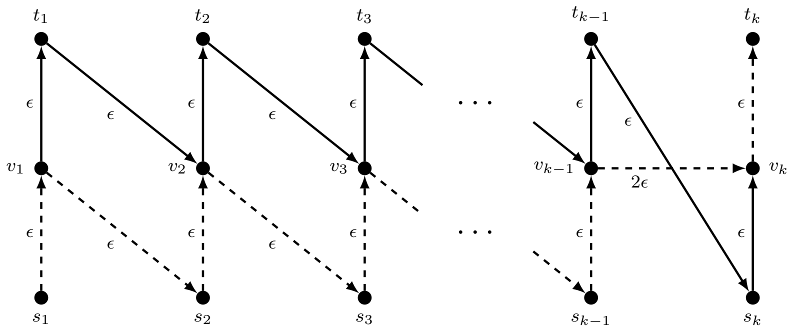

Fig. 4 Instance for Theorem 4.

Theorem 4 $\mathrm{gap}^{\mathrm{total}}(\mathrm{SPR}^{\max}, \mathrm{SPR}) = \infty$.

Proof Consider the directed graph D in Figure 4. D has $3k$ nodes and $4k-2$ arcs, $k \in \mathbb{N}$. Based on D we construct a timetabling instance I as follows. We associate 2 lines with the arcs of D. There is one line (dashed arcs) from s_1 to t_k with a driving time of $\epsilon = \frac{1}{k}$ on each arc except the second last arc with a driving time of 2ϵ. The second line (solid arcs) starts in v_1 and ends in v_k. The driving time for each arc of this line equals ϵ. We set the passenger demand for each OD-pair (s_i, t_i), $1 \le i \le k$, to one and zero otherwise.

For each OD-pair $(s_i, t_i) \in \mathcal{D}$, there exists only a single path from s_i to t_i via the node v_i. And for each line activity in I the lower time bound equals the upper time bound and the dwell time equals zero at every station. Hence, both (SPR) and (SPR$^{\max}$) reduce to determining for both lines the departure time at the first station. Again, both lines are constructed in such a way that the transfer times at nodes v_i, $1 \le i \le k-1$, are all identical. And the transfer times at the nodes v_{k-1} and v_k sum up to at least $T - \epsilon$.

First consider problem (SPR$^{\max}$). In an optimal solution, the dashed line departs at s_1 at 0 and the solid line departs at v_1 at $\frac{T+\epsilon}{2}$. The resulting transfer time for each OD-pair (s_i, t_i) at v_i is $\frac{T-\epsilon}{2}$. Hence, the total travel time for this timetable is $2k\epsilon + k\frac{T-\epsilon}{2} = \frac{1}{2}(3k\epsilon + kT)$.

In an optimal solution to (SPR), the departure time of the dashed line at s_1 is 0 and the solid line departs at v_1 at ϵ. The resulting transfer time for each OD-pair (s_i, t_i), $1 \le i \le k-1$, at v_i is zero and the transfer time at v_k equals $T - \epsilon$ for the pair (s_k, t_k). The total travel time for all passenger is therefore $2k\epsilon + T - \epsilon$.

We can conclude that

$$\frac{\tau^{\mathrm{total}}(\mathrm{SPR}^{\max}; I)}{\tau^{\mathrm{total}}(\mathrm{SPR}; I)} = \frac{3k\epsilon + kT}{2(2k\,\epsilon + T - \epsilon)} = \frac{3 + k\,T}{4 + 2T - \frac{2}{k}} \quad \underset{k\to\infty}{\longrightarrow} \quad \infty.$$

This finishes the proof. □

We finally give a Lemma that shows that there exists no instance such that the gap of the maximum weighted total travel time and the gap of the maximum weighted travel time among all passengers can both be arbitrarily large since they bound each other. Furthermore, the following lemma implies, that both gaps are bounded by the number of OD-pairs.

Lemma 1 *Let* $k := |\mathcal{D}| = |\{(s,t) \in V_{\text{dep}} \times V_{\text{arr}} : d_{st} > 0\}|$ *be the number of OD-pairs, then we have for every instance* $I \in \{\text{SPR}, \text{SPR}^{\max}\}$

$$\frac{\tau^{\text{total}}(\text{SPR}^{\max}; I)}{\tau^{\text{total}}(\text{SPR}; I)} \leq k \frac{\tau^{\max}(\text{SPR}^{\max}; I)}{\tau^{\max}(\text{SPR}; I)} \leq k$$

and

$$\frac{\tau^{\max}(\text{SPR}; I)}{\tau^{\max}(\text{SPR}^{\max}; I)} \leq k \frac{\tau^{\text{total}}(\text{SPR}; I)}{\tau^{\text{total}}(\text{SPR}^{\max}; I)} \leq k.$$

Proof Let $(x', y') \in \arg\max \tau^{\text{total}}(\text{SPR}^{\max}; I)$ be an optimal solution of instance I for problem $\text{SPR}^{\max}$ yielding the maximum total weighted travel time and $(x'', y'') \in \arg\max \tau^{\max}(\text{SPR}^{\max}; I)$ be an optimal solution yielding the maximum weighted travel time, i.e., by definition we have

$$\tau^{\text{total}}(\text{SPR}^{\max}; I) = \sum_{(s,t) \in \mathcal{D}} \sum_{p \in \mathcal{P}_{st}} \sum_{a \in p} d_{st}\, x'_a\, y'_p$$

and for an OD-pair $(s'', t'') \in \mathcal{D}$

$$\tau^{\max}(\text{SPR}^{\max}; I) = \sum_{p \in \mathcal{P}_{s''t''}} \sum_{a \in p} d_{s''t''}\, x''_a\, y''_p.$$

Since (x', y') and (x'', y'') give both rise to an optimal solution of $(\text{SPR}^{\max}; I)$, there exists an OD-pair $(s', t') \in \mathcal{D}$ such that

$$\sum_{p \in \mathcal{P}_{s''t''}} \sum_{a \in p} d_{s''t''}\, x''_a\, y''_p = \sum_{p \in \mathcal{P}_{s't'}} \sum_{a \in p} d_{s't'}\, x'_a\, y'_p.$$

Hence, we get

$$\begin{aligned}
\tau^{\text{total}}(\text{SPR}^{\max}; I) &= \sum_{(s,t) \in \mathcal{D}} \sum_{p \in \mathcal{P}_{st}} \sum_{a \in p} d_{st}\, x'_a\, y'_p \\
&\leq \sum_{(s,t) \in \mathcal{D}} \sum_{p \in \mathcal{P}_{s't'}} \sum_{a \in p} d_{s't'}\, x'_a\, y'_p \\
&= k \sum_{p \in \mathcal{P}_{s't'}} \sum_{a \in p} d_{s't'}\, x'_a\, y'_p \\
&= k \sum_{p \in \mathcal{P}_{s''t''}} \sum_{a \in p} d_{s''t''}\, x''_a\, y''_p \\
&= k\, \tau^{\max}(\text{SPR}^{\max}; I).
\end{aligned}$$

Similarly, we can argue $\tau^{\text{total}}(\text{SPR}; I) \geq \tau^{\max}(\text{SPR}; I)$ and conclude

$$\frac{\tau^{\text{total}}(\text{SPR}^{\max}; I)}{\tau^{\text{total}}(\text{SPR}; I)} \leq \frac{k\,\tau^{\max}(\text{SPR}^{\max}; I)}{\tau^{\text{total}}(\text{SPR}; I)} \leq k \frac{\tau^{\max}(\text{SPR}^{\max}; I)}{\tau^{\max}(\text{SPR}; I)}.$$

□

5 Computations

The aim of this section is to also give some computational evidence that routing decisions do indeed have a significant impact on timetabling. To this purpose, we compare the solution of an integrated timetabling and shortest path routing model (SPR) with a fixed passenger routing resulting from a real-world reference timetable.

We consider a scenario from a cooperation with the public transit company of Wuppertal, the Wuppertaler Stadtwerke (WSW), which is operating the famous cableway line "Schwebebahn". The data represents the periodic timetable of the core network of the public transport system of Wuppertal for the year 2013. The network has 158 station nodes, 229 OD-nodes, and 460 directed arcs. There are 71 lines: 67 bus lines, three city train lines, and the cableway line. The lines are operated at different frequencies; their period times are 10, 15, 20, 30, or 60 minutes. The data also contains the connections to the regional railway system, such that we can take these important transfers into account. After some preprocessing, the data contains 45 254 OD-pairs with a positive demand (we remove all OD-pairs for which the shortest connection for any timetable does not contain a transfer). Furthermore, we assume that each transfer has a lower time bound of 2 minutes.

For the computations, we use a time-expanded version of our integer programming model (SPR) that integrates a passenger routing. It works roughly as follows. We introduce for each line a binary variable representing the departure time at its first station. The passengers are represented by a path-flow in a time expanded network, in which they can travel freely. In the fixed routing case the demand of each OD-pair is sent along some shortest path w.r.t. a given reference timetable, namely, the WSW timetable of 2013 (WSW2013). The objective is to minimize the total weighted travel time. The core network of Wuppertal gives rise to a time-expanded event-activity network with 86 386 events and 431 604 activities. There are 3 990 binary line variables modeling the timetable. The passenger path-flow variables are dynamically added with a column generation algorithm, solving shortest path pricing problems. Our code is based on the constraint integer programming framework SCIP version 3.1.0 using Cplex 12.6 as an LP-solver. All computations were done on an Intel(R) Xeon(R) CPU E3-1245, 3.4 GHz computer (in 64 bit mode) with 8 MB cache, running Linux and 32 GB of memory. We set the time limit to 12 hours.

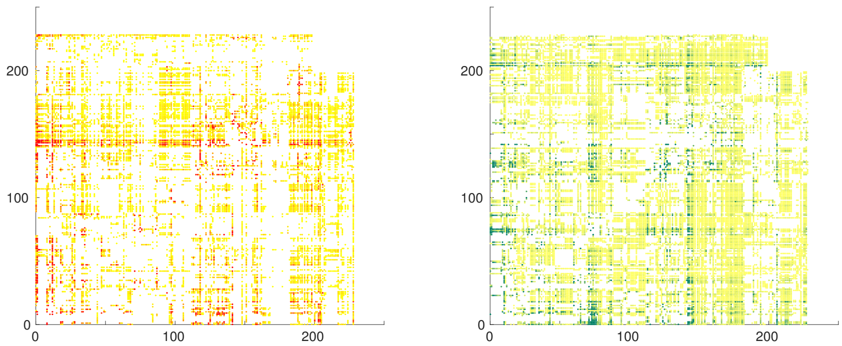

Fig. 5 Heat maps comparing differences in travel times between timetables computed with different passenger routing models. The axes of both diagrams correspond to the OD-nodes. The color of a point represents the difference in the travel time for the corresponding OD-pair between the best passenger routing for WSW2013 reference timetable and the result of an integrated timetable and passenger routing optimization. *Left:* The redder a dot the better is the travel time for the timetable computed with the fixed routing. *Left:* The greener a dot the better is the travel time for the timetable computed with the shortest path routing.

The WSW2013 reference timetable results in a total weighted travel time of 2 630 211.97 minutes and a total weighted transfer waiting time of 171 985.41 minutes. Fixing this routing and optimizing a classical PESP model, we could not find a timetable that improves the total weighted travel time. With the integrated timetabling and passenger routing model (SPR), however, we found a timetable that yields a total weighted travel time of 2 597 571.95 minutes and a total weighted transfer waiting time of only 131 456.07 minutes. This corresponds to an improvement of 1.24% in travel time and 23.57% in transfer waiting time. While the first improvement is marginal, the latter is substantial, in particular, since transfer waiting time is known to be perceived beyond proportion by passengers. The solution still has an optimality gap of 12%. Figure 5 illustrates the worsening and the improvement of the travel time for each OD-pair when comparing the passenger routings arising from the reference timetable and an integrated timetable and passenger routing optimization. The figure shows that for the integrated solution the number of OD pairs where the travel time decreases is much larger than the number of OD pairs where the travel time increases compared to the reference solution.

6 Conclusion

In this paper we investigated the influence of different passenger routing variants on timetable optimization. We showed that the best timetable for a fixed or lower bound routing can yield total travel times that are arbitrarily larger than an optimal timetable, i.e., a timetable optimized w.r.t. an integrated passenger routing. If we do not consider capacity constraints then all passengers can be assumed to use the same shortest path. If line capacities have to be fulfilled we showed that the total travel times can be reduced if the passengers of one OD pair are allowed to split their travel routes. Finally, we showed that the maximum travel time of a timetable minimizing the total travel time is bounded by the number of OD pairs times the maximum total travel time of a timetable that minimizes the maximum total travel time. We implemented a time expanded model similar to the one of Kinder (2008) to compute a timetable with integrated passenger routing. First computational experience for data from the city of Wuppertal indicates that the total transfer waiting time can be substantially reduced by around 24% in comparison to a real-world reference solution.

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