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Evaluating the Quality of Railway Timetables from Passenger Point of View

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Abstract In the railway passenger service planning, the main focus is often on the feasibility of the solutions and/or the associated costs of the Train Operating Company (TOC). The costs of TOCs' are the driver for the non-cyclic version of the Train Timetabling Problem (TTP), whereas feasibility is the main concern of the cyclic version of the same problem. Usually, the passengers for whom the service is designed are not taken into consideration, when creating the timetables. This could be one of the main reasons for which the willingness of passengers to use trains as their mean of transport has reduced. In this research, a choice based optimization approach is introduced that addresses this issue from passenger satisfaction point of view. We validate our model using a semi-real data of a major European railway company.

Keywords Railway Passenger Service · Ideal Timetable · Cyclic vs. Non-Cyclic

1 Introduction

In this study, we give attention to the problematic of providing the passenger service in railways. The offered product in this case is the timetable and the consumer is the passengers. However, the passenger demand is subject to the human behavior that incorporates several factors, to list a few: sensitivity to the time of the departure related to the trip purpose (weekday peak hours for work or school, weekends for leisure, etc.), comfort, perception and others. Moreover the passenger service has to compete with other transportation

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modes (car, national air routes, etc.) and thus faces even higher pressure to create good quality timetables.

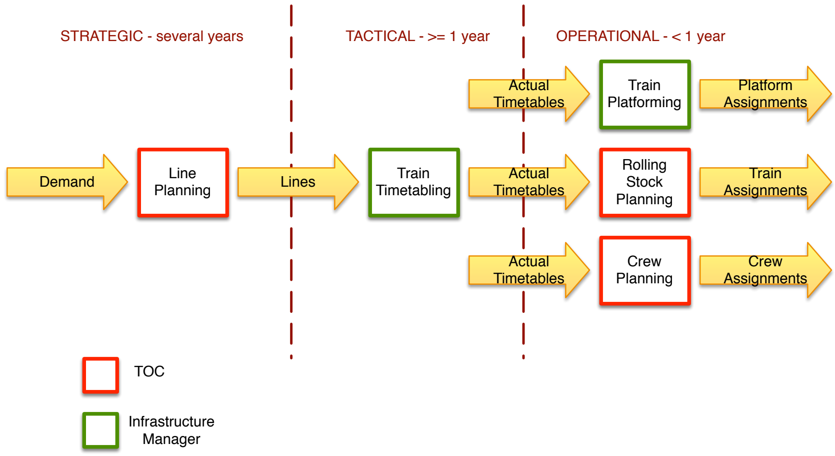


Fig. 1 *Planning overview of railway operation*

The current TOC planning horizon as described by Caprara et al (2007) is visualized on Figure 1. In the first stage, the Line Planning Problem (LPP) decides on which lines will be operated and with what frequencies. The LPP is the only problem in the planning horizon that actually takes into account the demand in the form of an hourly Origin Destination (OD) flows (Schöbel (2012)). The second stage in the planning horizon, is the Train Timetabling problem (TTP), where two different models exist: non-cyclic (Caprara et al (2002)) and cyclic (Peeters (2003)). The non-cyclic TTP takes as input the ideal timetables and tries to resolve the track conflicts by minimizing the timetable shifts needed. However, the origin of the ideal timetables is unclear as well as the punishment for the shifts. Similarly, in the cyclic problem, the model takes as an input the cycle and creates either arbitrary feasible solutions or a feasible solutions based on user defined objective function. Typically, the user defined functions are rather simplified such as minimization of travel time, which can not properly account for the passenger behavior. Both of the models can secure connections between two trains, however with no incentive if the connection is actually needed as none of the models takes passengers into account.

In the surveyed literature, a model that integrates mode choice and cyclic version of the TTP is presented (Cordone and Redaelli (2011)). The objective function is maximization of the demand captured by the railway mode as opposed to other modes. The constraint that estimates the demand captured by the timetable is using a logit model, whose attributes form the total trip length. The resulting formulation is non-linear and non-convex and is solved using heuristics.

In our study, we propose to integrate the Route Choice Model (RCM) and the TTP forming a new planning phase called the Ideal Train Timetabling Problem (ITTP). The ITTP is using the output of the LPP and serves as an input to the traditional TTP and hence, it is placed between the two respective problems (Figure 2). In this problem, we mimic the RCM by introducing a passenger cost related to a concerned timetable. The objective function of this problem is the passenger cost minimization. The model will allow timetables of the TOC's train lines to take the form of the non-cyclic or cyclic schedule. Moreover, we introduce a demand induced connections. The connections between the trains are not pre-defined, but are subject to the demand (via passengers' costs).

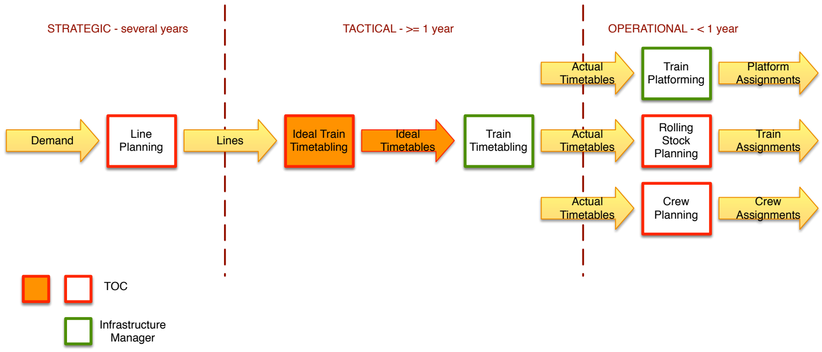


Fig. 2 *Modified overview of railway operation*

The structure of the manuscript is as follows: we introduce a definition of a passenger cost (Section 2), followed by a problem definition and its mathematical formulation (Section 3). The model is tested on a Swiss case study (Section 4). The paper is finalized by drawing some conclusions and discussion of possible extensions (Section 5).

2 Passenger Cost

In order to find a good timetable from the passenger point of view, we need to take into account passenger behavior. Such a behavior can be modeled using discrete choice theory (Ben-Akiva and Lerman (1985)). The base assumption in discrete choice theory is that the passengers maximize their utility, *i.e.* minimize the cost associated with each alternative and select the best one.

We propose the following costs associated with ideal passenger timetable:

– in-vehicle-time (VT)

– waiting time (WT)
– number of transfers (NT)
– scheduled delay (SD)

The **in-vehicle-time** is the (total) time passengers spend on board of (each) train. This time allows the passengers to distinguish between the "slow" and the "fast" services.

The **waiting time** is the time passengers spend waiting between two consecutive trains in their respective transfer points. The cost perception related to the waiting time is evaluated as double and a half of the in-vehicle-time (see Wardman (2004)).

The **transfer(s)** aim at distinguishing between direct and interchange services. In literature and practice, it is by adding extra travel (in-vehicle) time to the overall journey. In our case, we have followed the example of Dutch Railways (NS), where penalty of 10 minutes per transfer is applied (see de Keizer et al (2012)). Even though variety of studies show that number of interchanges, distance walked, weather, *etc.* play effect in the process, it is rather difficult to incorporate in optimization models. Thus using the applied value (by NS) will bring this research closer to the industry.

The **scheduled delay** is indicating the time of the day passengers want to travel, *i.e.* following the assumption that the demand is time dependent. For example: most of the people have to be at their workplace at 8 a.m. Since it is impossible to provide service that would secure ideal arrival time to the destination for everyone, scheduled delay functions are applied (Figure 3).

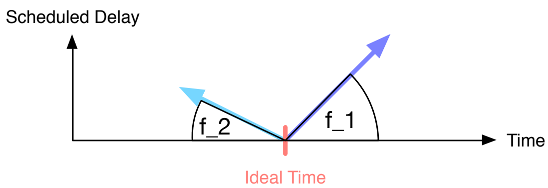


Fig. 3 *Scheduled Delay Functions*

As shown in Small (1982), the passengers are willing to s hift their arrival time by 1 to 2 minutes earlier, if it will save them 1 minute of the in-vehicle-time, similarly they would shift their arrival by 1/3 to 1 minute later for the same in-vehicle-time saving. If we would consider the boundary case, the lateness ($f_1 = 1$) is perceived equal to the in-vehicle-time and earliness ($f_2 = 0.5$) has half of the value (as seen on Figure 3).

To estimate the perceived cost (quality) of the selected itinerary in a given timetable for a single passenger, we sum up all the characteristics:

$$C = VT + 2.5 \cdot WT + 10 \cdot NT + SD \, [\text{min}] \tag{1}$$

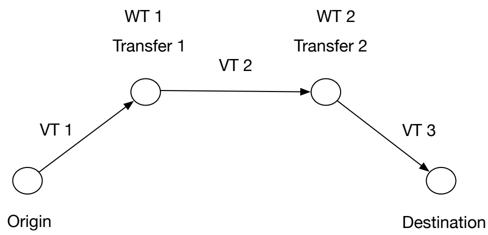


Fig. 4 *Example Network*

For a better understanding, consider the following example using network on Figure 4: passenger's itinerary consists of taking 3 consecutive trains in order to go from his origin to his destination, he has to change train twice. If he arrives to his destination earlier than his ideal time, his SD will be:

$$SD_e = \text{argmax}\left(\frac{ideal\ time - arrival\ time}{2}\ ,\ 0\right) \tag{2}$$

We use argmax function as one train line has several trains per day scheduled and the passenger selects the one closest to his desired traveling time. On the other hand, if he arrives later than his ideal time, then his SD will be:

$$SD_l = \text{argmax}\,(0\ ,\ arrival\ time - ideal\ time) \tag{3}$$

The overall scheduled delay is then formed:

$$SD = \text{argmin}\,(SD_e\ ,\ SD_l) \tag{4}$$

His overall perceived cost will be the following:

$$C = \sum_{trains} VT + 2.5 \cdot \sum_{transfers} WT + 10 \cdot NT + SD\,[\text{min}] \tag{5}$$

The resulting value is in minutes, however it is often desirable to estimate the cost in monetary values for pricing purposes. In such a case, national surveys estimating respective nation's value of time (VOT) exist. The VOT is given in nation's currency per hour, for instance in Switzerland the VOT for commuters using public transport is 27.81 swiss francs per hour (Axhausen et al (2008)). To make the cost in monetary units, simply multiply the whole Equation 1 by the VOT/60.

The aim of our research is not to calibrate the weights in Equation 1, but to provide better timetables in terms of the departure times. The weights serve as an input for our problem and thus can be changed at any time. Adding everything up, the ideal passenger timetable can be defined as follows:

> The ideal passenger timetable consists of train departure times that passengers' global costs are minimized, *i.e.* the most convenient path to go from an origin to a destination traded-off by a timely arrival to the destination for every passenger.

Similar concept, improving quality of timetables has been done in Vansteenwegen and Oudheusden (2006, 2007). Their approach has been focused on reliable connections for transferring passengers, whereas in our framework we focus on the overall satisfaction of every passenger.

Other concept similar to ours has been used in the delay management, namely in Kanai et al (2011) and Sato et al (2013). However their definition of dissatisfaction of passengers omits the scheduled delay.

3 Mathematical Formulation

In this section, we present a mixed integer programming formulation for the Ideal Train Timetabling Problem. The aim of this problem is to provide the ideal timetables, *i.e.* to minimize the passenger cost. The input of the ITTP is the demand that takes the form of the amount of passengers that want to travel between OD pair $i \in I$ and that want to arrive to their destination at their ideal time $t \in T_i$. Apart of that, there is a pool of lines $l \in L$ and its segments $g \in G^l$. Segment is a part of the line between two stations, where the train does not stop. Each line has an assigned frequency expressed as the available trains $v \in V^l$ (lines, segments and frequencies are the output of the LPP). Based on the pool of lines, the set of paths between every OD pair $p \in P_i$ can be generated. The path is called an ordered sequence of lines to get from an origin to a destination including details such as the running time from the origin of the line to the origin of the OD pair h_i^{pl} (where $l = 1$), the running time from an origin of the OD pair to a transferring point between two lines r_i^{pl} (where $l = 1$), the running time from the origin of the line to the transferring point in the path h_i^{pl} (where $l > 1$ and $l < |L^p|$), the running time from one transferring point to another r_i^{pl} (where $l > 1$ and $l < |L^p|$) and the running time from the last transferring point to a destination of the OD pair r_i^{pl} (where $l = |L^p|$). Note that the index p is always present as different lines using the same track might have different running times.

Part of the ITTP is the routing of the passengers through the railway network. Using a decision variable x_i^{tp}, we secure that each passenger (combination of indices it) can use at most one path. If there is no path assigned to a given passenger (due to the limited capacity of the trains), it is assumed that the passenger would take the earliest possible shortest path outside of the planning horizon H.

Within the path itself, passenger can use exactly one train on every line in the path (decision variable y_i^{tplv}). These decision variables, among others, allow us to backtrace the exact itinerary of every passenger. The timetable is understood as a set of departures for every train on every line (values of d_v^l). The timetable can take form of a non-cyclic or a cyclic version (depending if the cyclicity constraints are active, see below).

Since we know the exact itinerary of every passenger, we can measure the train occupation o_{vg}^l of every train v of every line l on each of its segment g. Derived from the occupation, number of train units u_v^l is assigned to each train. This value can be equal to zero, which means that the train is not running and the frequency of the line can be reduced.

We can formulate the ITTP as follows:

Sets Following is the list of sets used in the model:

I	–	set of origin-destination pairs
T_i	–	set of ideal times for OD pair i
P_i	–	set of possible paths between OD pair i
L	–	set of operated lines
L^p	–	set of lines in the path p
V^l	–	set of available trains for the line l (frequency)
G^l	–	set of segments on line l

Input Parameters Following is the list of parameters used in the model:

H	–	end of the planning horizon [min]
M	–	sufficiently large number (can take the value of H)
m	–	minimum transfer time [min]
c	–	cycle [min]
π_i^t	–	ideal arrival time of a passenger it to his destination [min]
r_i^{pl}	–	running time between OD pair i on path p using line l [min]
h_i^{pl}	–	time to arrive from the starting station of the line l to the origin/transferring point of the OD pair i in the path p [min]
D_i^t	–	demand between OD pair i with ideal time t [passengers]
q	–	value of the in vehicle time [monetary units per minute]
q_w	–	value of the waiting time in the relation to the VOT [unitless]
f_1	–	coefficient of being late in the relation to the VOT [unitless]
f_2	–	coefficient of being early in the relation to the VOT [unitless]
a	–	penalty for having a train transfer [min]
β	–	capacity of a single train unit [passengers]
j	–	maximum length of the train [train units]
γ_i	–	in-vehicle-time of the shortest path between OD pair i [min]
η_i	–	number of transfers in the shortest path for OD pair i [unitless]
$\overline{C}_i^t$	–	penalty cost for not serving passenger it inside of the planning horizon H [monetary units]

Decision Variables Following is the list of decision variables used in the model:

$\mathcal{C}_i^t$	–	the total cost of a passenger with ideal time t between OD pair i

w_i^t	–	the total waiting time of a passenger with ideal time t between OD pair i
w_i^{tp}	–	the total waiting time of a passenger with ideal time t between OD pair i using path p
w_i^{tpl}	–	the waiting time of a passenger with ideal time t between OD pair i on the line l that is part of the path p, i.e. the waiting time in the transferring point, when transferring to line l
x_i^{tp}	–	1 – if passenger with ideal time t between OD pair i chooses path p; 0 – otherwise
s_i^t	–	the value of the scheduled delay of a passenger with ideal time t between OD pair i
s_i^{tp}	–	the value of the scheduled delay of a passenger with ideal time t between OD pair i traveling on the path p
d_v^l	–	the departure time of a train v on the line l (from its first station)
y_i^{tplv}	–	1 – if a passenger with ideal time t between OD pair i on the path p takes the train v on the line l; 0 – otherwise
z_v^l	–	dummy variable to help modeling the cyclicity corresponding to a train v on the line l
o_{vg}^l	–	train occupation of a train v of the line l on a segment g
u_v^l	–	number of train units of a train v on the line l
α_v^l	–	1 – if a train v on the line l is being operated; 0 – otherwise

Routing Model The ITTP model can be decomposed into 2 parts: routing and cost estimation. The routing takes care of the feasibility of the solution, whereas cost estimation takes care of the passenger cost attributes. At first, we present the Routing Model (RM):

$$\min \sum_{i \in I} \sum_{t \in T_i} D_i^t \cdot \mathcal{C}_i^t \tag{6}$$

$$\sum_{p \in P_i} x_i^{tp} \leq 1, \qquad \forall i \in I, \forall t \in T_i, \tag{7}$$

$$\sum_{v \in V^l} y_i^{tplv} = x_i^{tp}, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p, \tag{8}$$

$$\left(d_v^l - d_{v-1}^l\right) = c \cdot z_v^l, \qquad \forall l \in L, \forall v \in V^l : v > 1, \tag{9}$$

$$o_{vg}^l = \sum_{i \in I} \sum_{t \in T_i} \sum_{p \in P_i} y_i^{tplv} \cdot D_i^t, \qquad \forall l \in L, \forall v \in V^l, \forall g \in G^l, \tag{10}$$

$$u_v^l \cdot \beta \geq o_{vg}^l, \qquad \forall l \in L, \forall v \in V^l, \forall g \in G^l, \tag{11}$$

$$\alpha_v^l \cdot j \geq u_v^l, \qquad \forall l \in L, \forall v \in V^l, \tag{12}$$

$$\mathcal{C}_i^t \geq 0, \qquad \forall i \in I, \forall t \in T_i, \tag{13}$$

$$d_v^l \geq 0, \qquad \forall l \in L, \forall v \in V^l, \tag{14}$$

$$x_i^{tp} \in (0, 1), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{15}$$

$$y_i^{tplv} \in (0,1)\,, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i,$$
$$\forall l \in L^p, \forall v \in V^l, \tag{16}$$
$$o_{vg}^l \geq 0, \qquad \forall l \in L, \forall v \in V^l, \forall g \in G^l, \tag{17}$$
$$u_v^l \in (0,j)\,, \qquad \forall l \in L, \forall v \in V^l, \tag{18}$$
$$\alpha_v^l \in (0,1)\,, \qquad \forall l \in L, \forall v \in V^l, \tag{19}$$
$$z_v^l \in \mathbb{N}, \qquad \forall l \in L, \forall v \in V^l. \tag{20}$$

The objective function **(6)** aims at minimizing the passenger cost. Constraints **(7)** secure that every passenger is using at most one path to get from his/her origin to his/her destination. Similarly constraints **(8)** make sure that every passenger takes exactly one train on each of the lines in his/her path, if this path is being used. Constraints **(9)** model the cyclicity using integer division. When solving the non-cyclic version of the problem, these constraints have to be removed. Constraints **(10)** keep track of a train occupation. Constraints **(11)** verify that the train capacity is not exceeded on every stretch/segment of the line. Constraints **(12)** assign train drivers, *i.e.* if a train v on the line l is being operated or not. Constraints **(13)**–**(20)** set the domains of decision variables.

Cost Estimating Constraints To make the ITTP complete, we need to expand the Routing Model with the cost estimating constraints. We will add the cost related constraints in blocks of attributes that create the cost of a passenger.

$$s_i^t \geq s_i^{tp} - M \cdot \left(1 - x_i^{tp}\right), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{21}$$

$$s_i^{tp} \geq f_1 \cdot \left(\left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right) - \pi_i^t\right)$$
$$-M \cdot \left(1 - y_i^{tp|L|v}\right), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall v \in V^{|L|}, \tag{22}$$

$$s_i^{tp} \geq f_2 \cdot \left(\pi_i^t - \left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right)\right)$$
$$-M \cdot \left(1 - y_i^{tp|L|v}\right), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall v \in V^{|L|}, \tag{23}$$

$$s_i^t \geq 0, \qquad \forall i \in I, \forall t \in T_i, \tag{24}$$
$$s_i^{tp} \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i. \tag{25}$$

The first block of constraints takes care of the **scheduled delay (SD)**. In our model we have 2 types of scheduled delay: SD for every path (constraints **(25)**) and SD that is linked to the path, which will be the final selected path of a given passenger(s) with a given ideal time (constraints **(24)**). As described in the Section 2, the constraints **(22)** model the earliness of the passengers (Equation **2**) and constraints **(23)** model the lateness (Equation **3**). Constraints **(21)** make sure that only one SD is selected (Equation **4**) –

not necessarily the lowest one as it depends on the cost of the whole itinerary (constraints **(33)**), *i.e.* the path with the smallest overall cost will be selected for the given OD pair with a given ideal time. These constraints also allow us to avoid the non-linearity in the estimation of the final passenger cost (constraints **(33)**).

$$w_i^t \geq w_i^{tp} - M \cdot \left(1 - x_i^{tp}\right), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{26}$$

$$w_i^{tp} = \sum_{l \in L^p \setminus 1} w_i^{tpl}, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{27}$$

$$\begin{aligned} w_i^{tpl} \geq \Big(\Big(d_v^l + h_i^{pl}\Big) - \Big(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\Big)\Big) \qquad & \forall i \in I, \forall t \in T_i, \forall p \in P_i, \\ -M \cdot \Big(1 - y_i^{tpl'v'}\Big) - M \cdot \Big(1 - y_i^{tplv}\Big), \qquad & \forall l \in L^p : l > 1, l' = l - 1, \\ & \forall v \in V^l, \forall v' \in V^{l'}, \end{aligned} \tag{28}$$

$$\begin{aligned} w_i^{tpl} \leq \Big(\Big(d_v^l + h_i^{pl}\Big) - \Big(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\Big)\Big) \qquad & \forall i \in I, \forall t \in T_i, \forall p \in P_i, \\ +M \cdot \Big(1 - y_i^{tpl'v'}\Big) + M \cdot \Big(1 - y_i^{tplv}\Big), \qquad & \forall l \in L^p : l > 1, l' = l - 1, \\ & \forall v \in V^l, \forall v' \in V^{l'}, \end{aligned} \tag{29}$$

$$w_i^t \geq 0, \qquad \forall i \in I, \forall t \in T_i, \tag{30}$$

$$w_i^{tp} \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{31}$$

$$w_i^{tpl} \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p. \tag{32}$$

The second block of constraints is modeling the **waiting time (WD)**. There are 3 types of waiting time: the final selected waiting time in the best path (constraints **(30)**), the total waiting time of every path (constraints **(31)**) and the waiting time at every transferring point in every path (constraints **(32)**). The constraints **(28)** and **(29)** are complementary constraints that model the waiting time in the transferring points in every path. In other words, these two constraints find the two best connected trains in the two train lines in the passengers' path. Constraints **(27)** add up all the waiting times in one path to estimate the total waiting time in a given path. Constraints **(26)** make sure that only one WT is selected (similarly as constraints **(21)** for SD).

$$\begin{aligned} \mathcal{C}_i^t \geq q \cdot q_w \cdot w_i^t + q \cdot a \cdot \sum_{p \in P} x_i^{tp} \cdot (|L^p| - 1) \\ + q \cdot \sum_{p \in P} \sum_{l \in L^p} r_i^{pl} \cdot x_i^{tp} + q \cdot s_i^t, \qquad \forall i \in I, \forall t \in T_i, \end{aligned} \tag{33}$$

$$\mathcal{C}_i^t \geq \left(1 - \sum_{p \in P_i} x_i^{tp}\right) \cdot \overline{C}_i^t, \qquad \forall i \in I, \forall t \in T_i, \tag{34}$$

$$\overline{C}_i^t = q \cdot q_w \cdot (m + c + \eta_i) + q \cdot a \cdot \eta_i + q \cdot \gamma_i + \\ \left(H + c + \gamma_i + a \cdot \eta_i - \pi_i^t\right) \cdot f_1 \cdot q, \qquad \forall i \in I, \forall t \in T_i. \tag{35}$$

At last, constraints **(33)** combine all the attributes together as in Equation **5** multiplied by the VOT. If a passenger *it* can not be served within the planning horizon, the constraints **(34)** become active and penalize the passenger with a cost associated to his shortest path realized with the first possible path outside of the planning horizon (in the next cycle) – constraints **(35)**.

4 Case Study

In order to test the ITTP model, we have selected the network of S-trains in canton Vaud, Switzerland as our case study. The reduced network is represented on Figure 5 (as of timetable 2014). We consider only the main stations in the network (in total 13 stations). A simple algorithm in Java has been coded, in order to find all the possible paths between every OD pair. The algorithm allowed maximum of 3 consecutive lines to get from an origin to a destination. The traveling times have been extracted from the Swiss Federal Railways' (SBB) website. The minimum transfer time between two trains has been set to 4 minutes.

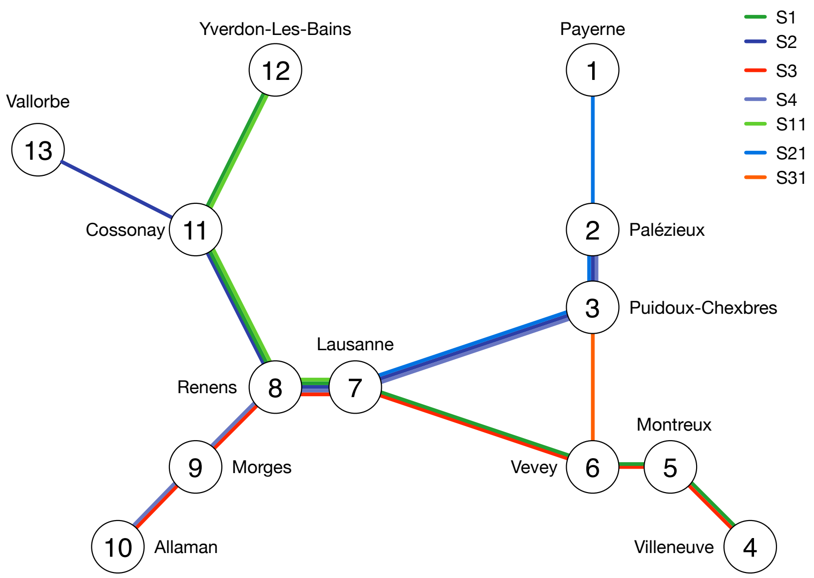


Fig. 5 *Network of S-trains in canton Vaud, Switzerland*

Line	ID	From	To	Departures			
S1	1	Yverdon-les-Bains	Villeneuve	–	6:19	7:19	8:19
	2	Villeneuve	Yverdon-les-Bains	5:24	6:24	7:24	8:24
S2	3	Vallorbe	Palézieux	5:43	6:43	7:43	8:43
	4	Palézieux	Vallorbe	–	6:08	7:08	8:08
S3	5	Allaman	Villeneuve	–	6:08	7:08	8:08
	6	Villeneuve	Allaman	–	6:53	7:53	8:53
S4	7	Allaman	Palézieux	5:41	6:41	7:41	8:41
	8	Palézieux	Allaman	–	6:35	7:35	8:35
S11	9	Yverdon-les-Bains	Lausanne	5:26*	6:34	7:34	8:34
	10	Lausanne	Yverdon-les-Bains	5:55	6:55	7:55	8:55
S21	11	Payerne	Lausanne	5:39	6:39	7:38*	8:39
	12	Lausanne	Payerne	5:24	6:24	7:24	8:24
S31	13	Vevey	Puidoux-Chexbres	–	6:09	7:09	8:09
	14	Puidoux-Chexbres	Vevey	–	6:31*	7:36	8:36

Table 4 *List of S-train lines in canton Vaud, Switzerland*

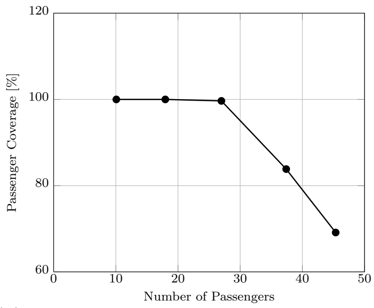
In Table 4, you can find the list of all S-train lines of the canton Vaud in the timetable of 2014. There are 7 lines that run in both directions. Each combination of a line and its direction has its unique ID number. Column "from" marks the origin station of the line as well as column "to" marks its destination. The columns "departures" show the currently operated timetable (*i.e.* departures from the origin of the line) in the morning peak hour (5 a.m. to 9 a.m.), which is the time horizon used in our study. Trains that did not follow the cycle (marked with a star *) were set to a cycle value, in order to not violate the cyclicity constraints (the timetables in Switzerland are cyclic with a cycle of one hour).

The SBB is operating the Stadler Flirt train units on the lines S1, S2, S3 and S4. In our case study, we have homogenized the fleet and thus use this type of a train also for the rest of the lines. The capacity of this unit is 160 seats and 220 standing people. The maximum amount of train units per train is 2 (as SBB never uses more units). The amount of train units per train remains the same along the line, but it might change at the end stations (we don't go into further details as this is the task of the Rolling Stock Problem).

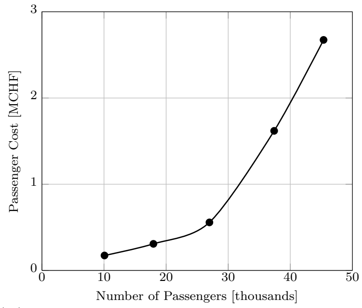
The demand and its distribution has been estimated based on the SBB report and observation (more details in Appendix A). In total there are 10 077 passengers in the network for the current situation. The coefficients of the passenger cost are as described in Section 2.

4.1 Results

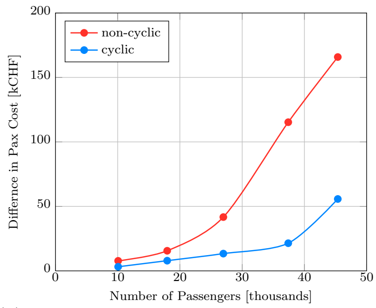
In all of the experiments, we have run 3 types of the ITTP model: current, cyclic and non-cyclic. The current model reflects the currently operated SBB timetable as in Table 4 (the decision variables d have been set to the values in the table). Subsequently, the cyclic model does not have the departure times as a hard constraint and thus the CPLEX can look for better values than those



(a) *Passenger coverage as a function of the demand for the current model*



(b) *Passenger cost as a function of the demand for the current model at ϵ 100*%*



(c) *The difference in passenger cost of the cyclic and non-cyclic model as compared to the current model at ϵ 100*%*

Fig. 6 *Results*

of the SBB. The non-cyclic model differs from the cyclic one by removing the cyclicity constraints. In order to speed up CPLEX, we would first solve the current version and give its solution as a warm start for the cyclic model and solve it. Further along, we would give the solution of the cyclic model as a warm start to the non-cyclic model.

Moreover, we have run the models for several levels of passenger density, starting from the real-like volume up until the point where the passenger coverage decreases to a level of 70%. The passenger coverage as a function of the demand for the current model (the coverage is more or less the same for the other two models) can be found on Figure 6(a). As it can be seen, the congestion starts at the amount of cca. 27 000 passengers and that the coverage goes down almost linearly.

The total passenger cost growth can be observed on Figure 6(b) (we plot only the current model as the other two models yield similar values). The passenger cost grows rather exponentially and its function can be split into two linear parts: non-congested (gradual slope) and congested (steep slope). This might be useful for practitioners as it would allow them to predict the passenger cost. Subsequently, we plot the relative difference of cyclic and non-cyclic timetables as opposed to the current timetable on Figure 6(c). In general, the cyclic model tends to find slightly better timetables than the current model (in the congested cases the benefit even dramatically increases). The non-cyclic timetable, on the other hand, is more flexible and copies the function of the total passenger cost (Figure 6(b)) and achieves more significant savings. This is due to the fact that the trains do not have to follow the cyclic frequency and thus are more densely scheduled, for instance in the most congested case, the average headway between two consecutive trains on a same line is 22.6 minutes, with minimum value of 1 minute and maximum value of 238 minutes.

5 Conclusions and Future Work

In this research, we define a new way, how to measure the quality of a timetable from the passenger point of view and introduce a definition of an ideal timetable. We then present a formulation of a mixed integer linear problem that can design the ideal timetables. The new Ideal Train Timetabling Problem fits into the current planning horizon of railway passenger service and is in line with the new market structure and the current trend of putting passengers back into consideration, when planning a railway service.

The novel approach not only designs timetables that fit the best the passengers, but that also creates by itself connections between two trains, when needed. Moreover, the output consists of the routing of the passengers and thus the train occupation can be extracted and be used efficiently, when planning the rolling stock assignment (*i.e.* the Rolling Stock Planning Problem). The ITTP can create both non-cyclic and cyclic timetables.

We test the model on a semi-real data of the S-train network of Canton Vaud in Switzerland. Our model was able to find a better timetable compared to the current SBB timetable, where the achieved savings, whilst keeping the timetables cyclic, were around 3 000 CHF and around 7 000 CHF, in the case of the non-cyclic timetable. Furthermore, we have focused on exploiting the passenger congestion. Our study shows that two linear functions, for congested and uncongested network, can be constructed and thus the passenger cost can be predicted. Most interestingly, we show that the improvements of the non-cyclic timetables as compared to the cyclic timetables, are flexible (they copy the total passenger cost functions) due to the fact that these timetables allow higher train density. The average train headway of the most congested case was reduced from the cycle (60 minutes) down to 22 minutes. Moreover the non-cyclic model was able to achieve around 160 000 CHF of savings even though the network is dense. These savings are expected to be even higher for less dense networks. Due to this fact, we would propose to combine the ITTP with the Line Planning Problem in the future.

In the future work, we will focus on efficient solving of the problem and extension of the planning horizon, *i.e.* to be able to solve the problem for a whole day. This would allow us to explore, if the non-cyclic timetables could perform better off-peak hours and in the context of the whole day. The new definition of a quality of a timetable (the passenger point of view) creates a lot of opportunities for future research: efficient handling of the TOC's fleet, better delay management, robust train timetabling passenger-wise or integration with other phases of the planning horizon.

References

Axhausen KW, Hess S, König A, Abay G, Bates JJ, Bierlaire M (2008) Income and distance elasticities of values of travel time savings: New swiss results. Transport Policy 15(3):173 – 185, DOI http://dx.doi.org/10.1016/j.tranpol.2008.02.001

Ben-Akiva M, Lerman S (1985) Discrete Choice Analysis. The MIT Press, Cambridge Massachusetts

Caprara A, Fischetti M, Toth P (2002) Modeling and solving the train timetabling problem. Operations Research 50(5):851–861, DOI http://dx.doi.org/10.1287/opre.50.5.851.362

Caprara A, Kroon LG, Monaci M, Peeters M, Toth P (2007) Passenger railway optimization. In: Barnhart C, Laporte G (eds) Handbooks in Operations Research and Management Science, vol 14, Elsevier, chap 3, pp 129–187

Cordone R, Redaelli F (2011) Optimizing the demand captured by a railway system with a regular timetable. Transportation Research Part B: Methodological 45(2):430 – 446, DOI http://dx.doi.org/10.1016/j.trb.2010.09.001, URL http://www.sciencedirect.com/science/article/pii/S0191261510001049

Kanai S, Shiina K, Harada S, Tomii N (2011) An optimal delay management algorithm from passengers' viewpoints considering the whole railway network. Journal of Rail Transport Planning & Management 1(1):25 – 37, DOI http://dx.doi.org/10.1016/j.jrtpm.2011.09.003, robust Modelling of Capacity, Delays and Rescheduling in Regional Networks

de Keizer B, Geurs K, Haarsman G (2012) Interchanges in timetable design of railways: A closer look at customer resistance to interchange between trains. In: AET (ed) Proceedings of the European Transport Conference, Glasgow, 8-10 October 2012 (online), AET

Peeters L (2003) Cyclic Railway Timetable Optimization. ERIM Ph.D. series Research in Management, Erasmus Research inst. of Management (ERIM)

Sato K, Tamura K, Tomii N (2013) A mip-based timetable rescheduling formulation and algorithm minimizing further inconvenience to passengers. Journal of Rail Transport Planning & Management 3(3):38 – 53, DOI http://dx.doi.org/10.1016/j.jrtpm.2013.10.007, robust Rescheduling and Capacity Use

Schöbel A (2012) Line planning in public transportation: models and methods. OR Spectrum 34:491–510, DOI 10.1007/s00291-011-0251-6

Small KA (1982) The scheduling of consumer activities: Work trips. The American Economic Review 72(3):pp. 467–479

Swiss Federal Railways (2013) SBB: Facts and Figures 2013. Tech. rep.
Vansteenwegen P, Oudheusden DV (2006) Developing railway timetables which guarantee a better service. European Journal of Operational Research 173(1):337 – 350, DOI http://dx.doi.org/10.1016/j.ejor.2004.12.013
Vansteenwegen P, Oudheusden DV (2007) Decreasing the passenger waiting time for an intercity rail network. Transportation Research Part B: Methodological 41(4):478 – 492, DOI http://dx.doi.org/10.1016/j.trb.2006.06.006
Wardman M (2004) Public transport values of time. Transport Policy 11(4):363 – 377, DOI http://dx.doi.org/10.1016/j.tranpol.2004.05.001

A Demand Generation

The total amount of passengers in the network has been estimated in the following manner: the population of Switzerland is 8 211 700 habitants and the population of Canton Vaud is 755 369 habitants, which leads to a rough ratio of 1:10. Applying this ratio to a reported amount of passenger journeys per day by SBB (in total one million for the whole SBB network), we arrive to a demand volume of 100 000 passenger journeys per day in canton Vaud. However not all of these journeys are being realized using S-trains. Since almost all of the trains in Canton Vaud have to pass through its capital city Lausanne, we can derive the ratio, between the S-trains and other class trains passing through Lausanne, of 40:60 percent, which leaves us with a 40 000 passenger journeys per day using S-trains in Canton Vaud. Furthermore, the SBB report provides hourly distribution of passengers on a regional services from Monday to Friday. According to this report 25 percent of the journeys are being realized in the morning peak hour, which gives us cca. 10 000 passenger journeys in the morning peak hour for the S-train network of Canton Vaud.

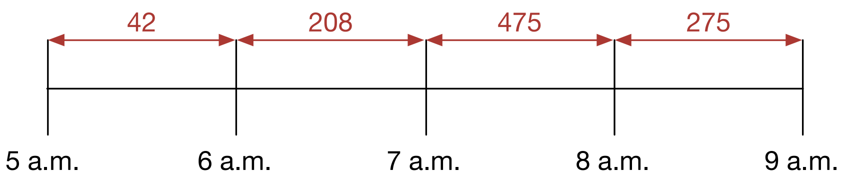


Fig. 7 *The hourly distribution of the passenger groups*

In order to ease the size of the generated lp file(s), the passengers have been split into 1 000 passenger groups (indices it) of varying sizes. These groups have been divided into hourly rates (Figure 7) according to the SBB report (Swiss Federal Railways (2013)) and smoothed into minutes using non-homogenous Poisson process. Since we use concept of an ideal arrival time to the destination, the generated arrival time at the origin has been shifted, by adding up the shortest path travel time between the OD pair, to the destination of the passengers.

In order to generate real-like OD flows (index i), we consider the following probabilities:

$p(D = 7) = 0.5$	–	probability of a destination being Lausanne
$p(D = 8) = 0.2$	–	probability of a destination being Renens
$p(D = other) = 0.3$	–	probability of a destination being other than Lausanne or Renens
$p(O = any) = 1/12$	–	probability of an origin being any station (except the already selected destination)

Since Lausanne is the biggest city in the Canton with all the lines, except the line 13 and 14, passing through it, it has the largest probability of being a destination (many people also use Lausanne as a transfer point to higher class trains). The city with the second highest probability is Renens, because it is the closest station to one of the biggest universities in Switzerland and from the network diagram (Figure 5), we can see the most of the lines stop there, which suggests high demand. The rest of the stations have equal probability of being a destination (0.3/11), which is rather small as in the morning peak hour people travel towards their work/school in big cities. On the other hand, the probability of being an origin is uniformly distributed and dependent on its destination (origin can not be the same as a destination). The final probability $p(O = o, D = d)$ for every OD pair can be seen in Table 6.

In order to reach the total demand, the average size of a group should be $\rho =$ the total demand divided by the number of groups. In the current scenario $\rho = 10\,000/1\,000 = 10$. In our study, we use 3 different classes of groups: small, medium and large. The size of the small group is drawn from the uniform distribution $\mathcal{U}(1, 0.6\rho)$ and applied to ODs with a probability $p(O = o, D = d) \in [0, 1.5)\,\%$. The size of the medium group follows $\mathcal{U}(0.6\rho+1, \rho)$ and is applied to ODs with a probability $p(O = o, D = d) \in [1.5, 3)\,\%$. The largest group size follows a distribution $\mathcal{U}(\rho+1, 2\rho)$ and is applied to a probability $p(O = o, D = d) \in [3, 4.5)\,\%$.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
2	0.002272727	0	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
3	0.002272727	0.002272727	0	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
4	0.002272727	0.002272727	0.002272727	0	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
5	0.002272727	0.002272727	0.002272727	0.002272727	0	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
6	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
7	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
8	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727
9	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0	0.002272727	0.002272727	0.002272727	0.002272727
10	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0	0.002272727	0.002272727	0.002272727
11	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0	0.002272727	0.002272727
12	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0	0.002272727
13	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.002272727	0.041666667	0.016666667	0.002272727	0.002272727	0.002272727	0.002272727	0

Table 6 *Origin – Destination distributions*