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Transit network design for small-medium size cities

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Abstract This paper aims to propose a novel heuristic to solve the network design problem for public transport.
Small-medium size cities can be defined as those with a diameter of few kilometers with up to few hundreds thousands residents. These urban centers have specific characteristics, functional and spatial mobility system. The transportation demand is widespread in origin and concentrated in a small number of attraction points close each other.
This particular structure of the demand ("many to few") in this small-medium size cities leads to define appropriate models and procedures, whose solution methods are developed around it, showing great results for the selected case study.
The variables of the model are the routes and their frequencies. The constraints of the problem affect the overall demand to be served, the quality of the proposed service (transfer, load factors) and the definition of routes.

Keywords: Transit network design, Small-medium size cities, Heuristic, Public Transport

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1 Introduction

Small-medium size cities can be defined as urban centers with a population of few hundred thousand inhabitants and restrained spatial dimensions (diameter of few kilometers). These cities have specific characteristics in terms of land use and mobility system, very often similar each other. This common characteristics, generally come out from actions and choices layered over time, can be described as follows:

- small historic center located in the central area of the city, fully built and with roads of limited width. This area contains the main part of city activities: the local administration and other various services, schools, hospitals, commercial and service activities. Thus, this area is characterized by the presence of both residences and jobs;
- radial structure for the other neighborhoods, generally for those more recently developed, which extend along the main roads connecting other neighbouring towns. These districts are reserved almost exclusively to the residence with limited presence of work activities. The building development is related to the distance from the center: as much farther as less densely built-up;
- important productive (industrial areas) and commercial areas (shopping centers) may also be present in the far periphery.

The urban structure described above affects the mobility system so that we can identify some characteristics of the mobility system:

- transport demand has a widespread in origin;
- transport demand is instead concentrated in a few major points of attraction, often located in the city center, such as administrative centers, school campuses, the railway station, the hospital and the production areas;
- the road network is spread radially and it is relatively sparsely connected;
- public transit has provided by buses with no lines of different types (i.e. main and feeder lines).

The above observations made on the urban structure and on the characteristics of the mobility system can be considered valid for a large number of small-medium sized urban centers.

2 State of the art

The design of public transport network, known as Transit Network Design (TND) problem is widely discussed in scientific literature. As the problem is complex and non-convex (Newell 1979; Baaj and Mahamassani 1991), the solution methods are usually formulated as a non-linear optimization problem with both discrete and continuous variables and constraints. The best and most efficient solution methods are based on heuristic procedures and meta-heuristic algorithms. A global review about route design, frequency setting, timetabling of transit lines, and their combination is proposed by Desaulniers and Hickman (2007), Guihaire and Hao

(2008), Kepaptsoglou and Karlaftis (2009). Many interesting solving procedures and algorithms have been proposed in literature. Some works are especially focused on the phase of the route generation: Baaj and Mahmassani (1995) with an Artificial-Intelligent heuristic algorithm for route generation; Carrese and Gori (2002) with a heuristic procedure for large urban areas with different categories of lines; Bagloee and Ceder (2011) with an heuristic procedure providing routes categorized by hierarchy (mass, feeder, local routes); Beltran et al. (2009) with an innovative application for the generation of routes operated with green vehicles; Mauttone and Urquhart (2009) with a Pair Insertion Algorithm (PIA) inspired by the Route Generation Algorithm (RGA) of Baaj and Mahmassani, where its original expansion of routes by inserting individual vertices is replaced by a strategy of insertion of pairs of vertices; Cipriani et al. (2012) with a route generation procedure based on the flow concentration process and a parallel genetic algorithm for finding a sub-optimal set of routes with the associated frequencies. Other works are instead focused on the implementation of efficient solution methods, especially based on updated meta-heuristic algorithms: Chakroborty (2003) working to highlight the effectiveness of using genetic algorithms for solving the urban TND problem; Fan and Mumford (2008) with an approach using hill-climbing and simulated annealing algorithms; Kuan et al. (2006) proposing the design and the analysis of two metaheuristics (genetic algorithms and ant colony optimization) for solving the feeder bus network design problem; Lownes and Machemehl (2010) providing a new mixed integer model for a single-route circulator design problem; Pattnaik et al. (1998) providing one of the first application of genetic algorithms; Szeto and Wu (2011) proposing a genetic algorithm hybridized with a neighborhood search heuristic to tackle the frequency setting problem; Yan et al. (2013) with a heuristic solution approach, based on k-shortest path algorithm, simulated annealing algorithm, Monte Carlo simulation, and probit type discrete choice model; Zhao and Zeng (2008) with metaheuristic search scheme that combines simulated annealing, tabu and greedy search methods. These procedures proposed in literature are usually working to address the solution in a many-to-many context in terms of transport demand rather than to a more characterized environment, like the one analyzed in this study. Such characteristic produces procedures able to solve TND problem in any possible context but, in many cases, this positive aspect is more than counterbalanced by very complex solution methods, not providing satisfying results in real network applications.

3 Problem definition

The problem can be formulated as an optimization problem introducing an objective function (OF) where the resources and the impacts of transport on the various stakeholders (operators and users) are measured.

The proposed formulation considers the total distance traveled by buses (in space and time), which determine the variable costs of the operations; the total number of vehicles used which determines the fixed costs; the disutility of users, composed of the in-vehicle time, the waiting time at bus stops and the inconvenience of potential transfers.

The objective function (OF) is therefore a weighted sum of the different transport costs:

$$C_1\sum_{i\in I_i} L_i \cdot f_i + C_2\sum_{i\in I_i}\sum_{hk\in I_{a,i}} tp_{hk,i}\cdot f_i + C_3 nb + C_4\sum_{i\in I_i}\sum_{hk\in I_{a,i}} tp_{hk}\cdot p_{hk,i} + C_5\sum_{i\in I_i}\sum_{hk\in I_{a,i}} ta_{hk,i}\cdot pa_{hk,i} + C_6\sum_{n\in I_n} nt_n$$

where:

$\sum_{i\in I_i} L_i \cdot f_i$	total traveled distance
$\sum_{i\in I_i}\sum_{hk\in I_{a,i}} tp_{hk,i}\cdot f_i$	total travel time
nb	number of used buses
$\sum_{i\in I_i}\sum_{hk\in I_{a,i}} tp_{hk}\cdot p_{hk,i}$	total users' in-vehicle time
$\sum_{i\in I_i}\sum_{hk\in I_{a,i}} ta_{hk,i}\cdot pa_{hk,i}$	total users' waiting time
$\sum_{n\in I_n} nt_n$	total number of transfers

The notations used are defined as follows:

L_i is the length of the line i (km), f_i is the frequency of line I (bus/h); I_i, $I_{a,i}$, I_n are respectively the set of lines, links and nodes; $tp_{hk,i}$, $ta_{hk,i}$ are respectively the travel time and waiting time for link hk of line i; $p_{hk,i}$, $pa_{hk,i}$ are respectively passengers on board and boarding passengers on the link hk of line i; nt_n is the number of passengers transferring on the node n; C_1, C_2, C_3, C_4, C_5, C_6 are monetary weights.

The input data include: the origin-destination matrix (O/D), the road network, the bus capacity and the network terminals. The constraints follow general guidelines to make the problem practically relevant and applicable in real world instances. In essence, they enforce: the total amount of demand to be served; the quality of service (maximum number of transfers equal to 1); serving all central attractors without transfers; avoiding too long or too short route lengths; maintaining the load factor lower than a predetermined threshold value. These will be formally defined whenever used in the proposed solving procedure in the next section.

4 Solving Procedure

As anticipated in the literature review, a proper formulation of a general instance of these kind of problems is extremely cumbersome and so it is its search for an optimal solution, which makes looking for optimality a good academic exercise potentially with relevant insights, but usually with little practical implications. The problem involves a simultaneous and combined solution of vehicle routing, assignment, facility location, lines' recombination and scheduling problems, many of which would be a challenging optimization problem to be solved even by itself in reasonable time for large enough instances.

Therefore, the proposed approach to solve the problem is a heuristic procedure. The goal is to provide a practical and fast algorithm to offer solutions that are better than current practices for the (many) urban areas with the characteristics described above.

The heuristics can be summarized with the following steps:

- localization of central and peripheral terminals;
- construction of the base network;
- expansion of the base network;
- connection of lines of the base network with all central attractors;
- lines frequency determination;
- linking of routes;
- iterative adjustment of lines frequency.

The description of the procedure is summarized in *Figure **1*** and in the steps described after.

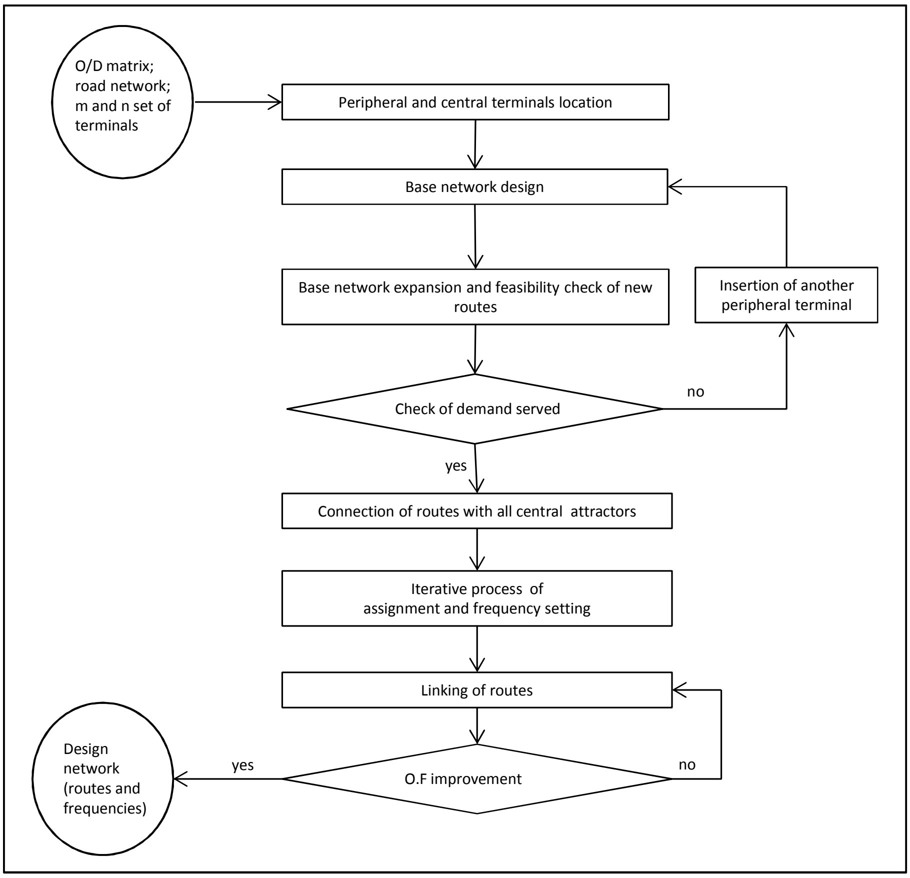

Figure 1: Solving procedure framework

Step 1 - Localization of terminals

First, we identify the major points of attraction and generation. Let *m* be a selection of the ones situated in the central area and *n* a selection of the ones situated in the peripheral area. The choice of *m* and *n* is left to the planner and can be changed during the procedure.

The number of terminals *m* and *n* is extremely important in terms of possible solutions generated. As it exceeds, respectively, 4-5 and 10-12 for central and peripheral ones, the large number of combinations between the peripheral and central terminals involves switching from a deterministic optimization procedure to a stochastic one.

Step 2 - Base network definition

The next step is to determine the base network linking each peripheral terminal to the nearest central terminal trough the shortest path. The steps of the algorithm are listed as follows:

- the selection of an external terminal;

- the generation of *m* paths from the selected peripheral terminal to each of the central ones, finding the shortest paths in terms of travel time;
- the shortest path is identified among all the *m* generated ones.

The operation is repeated for all the external terminals. This network represent the most efficient in order to satisfy the main part of transport demand, directed to the center and originated in the major peripheral areas.

Step 3 - Expansion of the base network

The base network must then be expanded in order to increase the total amount of demand served. The first step is to assign the transport demand to the transport network through an "All-Or-Nothing" technique assignment. Then, a systematic procedure of path modifications is tested for each previously generated routes. Such procedure involves the increase of a predetermined value (e.g. + 50%) the travel cost of the selected link. Afterwards, the new shortest path between the endpoints of the link selected is computed and the feasibility of the new route is verified according to these criteria:

- the increase in length of the route allows to serve new demand;
- the increase in length of the route is less than a predetermined threshold value (e.g. the 50%);
- the modification of the route is rational in terms of alignment. Each node has to be closer to the final destination than the previous one, even compared to other possible routes.

This last criterion is equivalent to verify both the two conditions, shown in *Figure 2*, which refers to the insertion in the route of node C between A and B. The insertion constraint is satisfied if a) the travel distance from B to D, final terminal of route, is smaller than that from C to D; b) the travel distance from C to D is smaller than that from A to D.

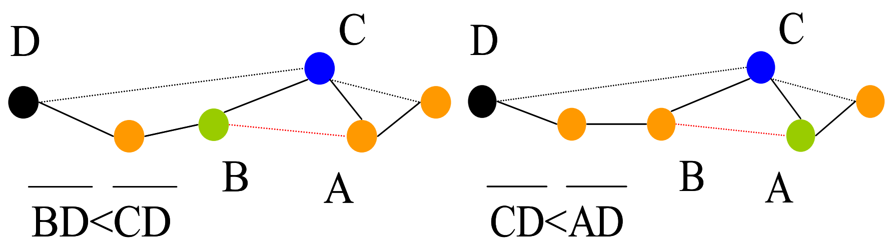

Figure 2: Replacement of link AB

The increase of the travel cost of the selected link enables to preserve the network connection. This condition could not be guaranteed if, as often used in other works (Mandl 1979; Pattnaik et al. 1998), the link is cut.

There are several possible selection criteria for the choice of link to be eventually replaced. The two criteria described as follows are the prevalent: 1) proceed in a sequential way by analyzing, route by route, all the links belonging to the route starting from one of the two terminals; 2) select among all the links of the network of routes, one at a time, starting from the one characterized by the minimum load of passengers.

The second criterion seems to be more efficient from a computational point of view: in fact, deleting from the routes, the links with smaller level of passengers could, for example, lead to satisfy the constraint on the minimum value of the demand to serve before checking all the links composing the network. The topology of the network and the distribution of the demand are the fundamental references in the choice of the selection rule of the links to be taken.

At the end of the expansion phase of the base network, the procedure aims to verify if the served demand is greater than a fixed threshold (x), for example 95%, through the following relationship:

$$\sum_{hk \in I_a} pa_{hk} - \sum_{n \in I_n} nt_n \geq x \sum_{ij \in I_{OD}} s_{ij}$$

where s_{ij} are the trips between nodes i and j belonging to the I_{OD} set.

If the constraint is not satisfied, a new iterative construction procedure is required by defining and expanding additional new routes. This phase is structured in the following steps:

- the selection of a new additional peripheral terminal;
- the increase of the travel cost of the links used at least from one route, by a predetermined factor, so limiting the overlap of routes;
- the generation of the new route using the shortest path to an internal terminal, and checking the possible expansions.

The steps are repeated adding new routes till to satisfy the constraint on the total amount of transport demand served.

Step 4 - Connecting routes to all central attractors

The connection of the routes to all central attractors consists in extending each route to the other central attractors still not reached by the route. The extension takes place in the direction which corresponds to the minimum total travel time of passengers of route i:

$$\min \sum_{hk \in I_{a,i}} tp_{hk,i} \cdot p_{hk,i}$$

The connection phase allows the transfer from any route to any other one, automatically satisfying the constraint of not having more than one transfer to reach any possible destination. In addition, through this step, it is possible to reach directly, without transfers, all the major attractors from any external terminals.

Step 5 – Frequency setting

After the fourth steps described, a first feasible solution (the base network) has been carried out and will be improved in the following steps. First of all, for the definition of the frequency, an iterative assignment and frequency setting procedure, is applied. At the beginning of the procedure is defined an initial set of frequencies. After the first assignment, based on the maximum load p_{max} recorded on the line, is estimated the frequency f_i taking into account the maximum permitted load factor on board and the capacity of the vehicle C_V :

$$f_i = \frac{p_{\max}}{fc_{\max} \cdot C_V}$$

The frequency thus obtained is then compared with the input one. The process ends if there are no significant variations between two iterations, otherwise a further assignment is required. The transport demand is loaded on the base network by means of a hyper-paths assignment procedure. The convergence of the iterative frequency setting procedure is not guaranteed, but all computational tests performed converged in few iterations.

Step 6 – Linking of routes

To further reduce the number of transfers is necessary to proceed to the linking of the routes, moving the final terminal from the center to the external terminals. This should also allow to limit, even if to a reduced extent, the overall length of the network, reducing the number of overlapping routes. The linking of the routes is not necessarily unique, since each route can be combined with more routes.

The linking of the routes is conducted through the following steps:

- the selection of the line that provides the greatest number of runs;
- the assessment of the path followed by the passengers of the selected line;
- the identification of the lines to or from which a transfer takes place;
- for each of the identified lines, the definition of the number of runs Ci to assign to the new line to be generated (union of the line identified with that selected in step 1 as a function of transfers between the lines):

  $$C_i = \left\lceil \frac{n_i}{\sum_{i \in I_{id}} n_i} \cdot C_s \right\rceil$$

  where Cs, n_i, I_{id} respectively represent the number of runs of the selected line s, the number of transfers between the line s and the line i and the set of the identified lines;

- for each of the lines identified which presents a number of runs *Ci* equal to or greater than one, the linking of the routes is carried out (an example of the combination of two routes is represented in
- **Figure** ***3***);
- the assignment and frequency setting procedure is repeated and the new OF value is calculated;
- the comparison of the OF value with the last solution at disposal and the acceptance of the linking of routes if there is an improvements of the OF value.

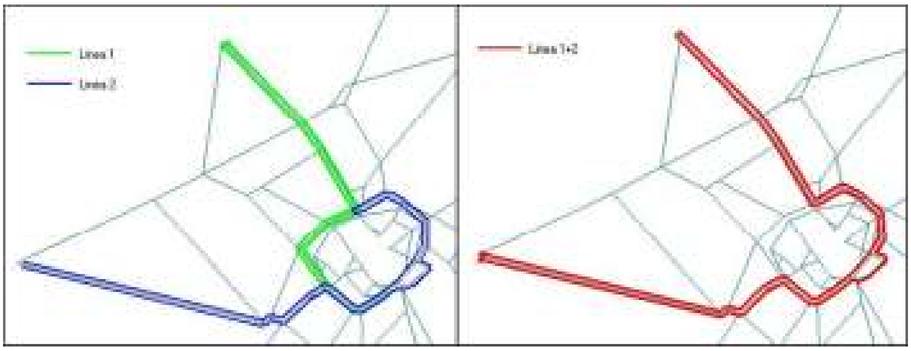

Figure 3: Linking lines

The algorithm iteratively repeats the sequence of operations for the remaining lines; these are always selected in descending order of number of runs. The process ends when all the routes carried out in the design network have been processed.

Step 7 – Frequency adjustment

Finally, the frequencies of the lines are then adjusted without modifying the routes following the same procedure described in step 5.

5 Real case study

The proposed procedure has been tested on a real network of a small–medium sized urban center. The aim is to verify the practical implementation of the procedure and to evaluate the performance of the proposed model. The choice of a real network has also enabled us to make a comparison between the results provided by the application of the proposed model and the public transit in use. The public transit network studied is the bus network of the town of Foligno, an urban center of about 55,000 inhabitants located in the Umbria region. The area is divided into 30 traffic zones. The O/D matrix on public transport consists of approximately 3,130 trips in the peak hour. The area of the old town and the external area with industrial activities represent the two major points of attraction and they are the destination of about 2/3 of the total transport demand (about 2,000 trips), while origins are distributed in all traffic zones.

The current public transit network consists of 16 conventional bus lines with fixed routes and frequencies. These lines connect the external areas, surrounding the center of Foligno, with the historic town center and the railway station; 5 lines run through the city center.

Figure 4 allows observing the ability of the procedure to ensure a good coverage of the urban area not only in terms of transport demand served but also in terms of links belonging to the road network itself. It is also important to observe that the relevant number of lines along the central ring ensures both the transfer opportunity among every line and the high number of runs available for those who travel in the central areas of the city.

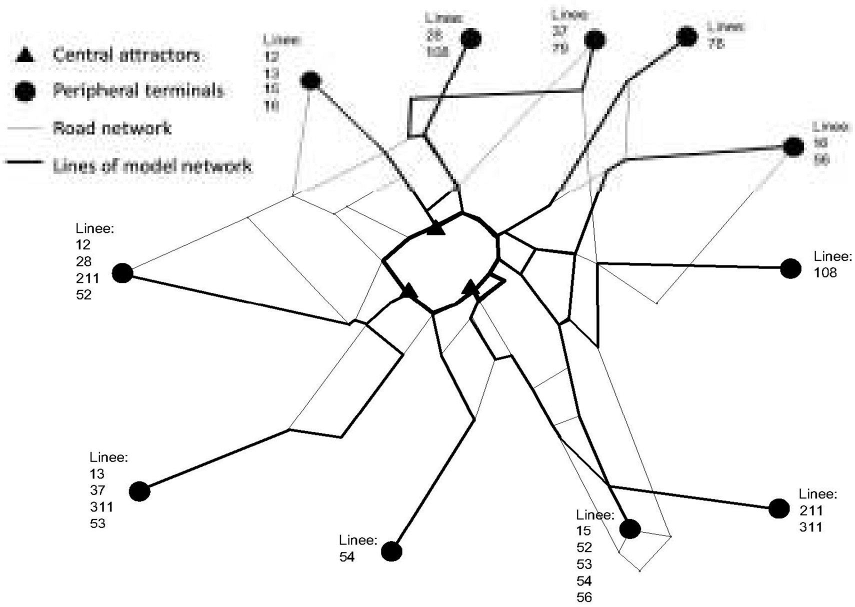

Figure 4: Final model network in the Foligno application

The following *Table 1*shows the values of the different components of the OF, computed from the results of the simulations carried out both on the design network and on the existing one. Regarding the design network, the table shows the data related to both the initial configuration of the network (step 4 - Connection line with all the main attractors) and the final configuration. The final comparison is carried out keeping fixed the total level of the supply offered by the bus services.

Indicator	Existing Network (a)	Base Network (b)	Final Network (c)	Difference (%) (b-c)	Difference (%) (c-a)
Number of lines	16	11	14	+21%	-12%
Buses-km	347	375	348	-7%	0%
Buses-h (min)	1015	1040	975	-6%	-4%
Number of buses	23	22	23	+4%	0%
Number of transfers	2710	1.260	860	-32%	-68%
Total in vehicle time (min)	33940	32625	32935	+1%	-3%
Total waiting time (min)	39460	37640	34310	-9%	-13%
Avg load factor	0.42	0.39	0.42	+8%	0%

Table 1: Performances comparison between existing and design network in the Foligno application

6 Conclusions

The proposed procedure provides encouraging results associated with a remarkable ease of execution on real networks. The ease of both the operation and the logic on which the algorithm is built, allows a complete knowledge of the procedure so as to facilitate the eventual intervention of the designer. The dependence on initial choices of parameters and constraints, typical of deterministic type of heuristics, is partly offset by the possibility of intervention during the procedure by the designer himself.

The proposed heuristic has been developed to solve small-medium size cities, due to their simpler network structure, which drove our choices in identifying proper steps. The heuristic is polynomial and suitable and capable to solve much larger size cities, whose more complex structure and characteristics could, however, undermine the underlying assumptions used to model our smaller cities.

Future research include: providing mathematical details and formulation of the constraints for the proposed model; testing and comparing the proposed procedure towards optimal values computed for small enough case studies.

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