
Effect of regenerative braking on energy-efficient train control

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Abstract An important topic to reduce the energy consumption in railways is the use of energy-efficient train control (EETC). Modern trains allow regenerative braking where the released kinetic energy can be reused. This regenerative braking has an effect on the optimal driving regimes compared to trains which can only use mechanical braking. This paper compares the impact of regenerative braking on the optimal train control strategy on level track. The energy-efficient train control problem is modelled as an optimal control problem over distance and solved using a Pseudospectral Method. Three different braking strategies are compared: mechanical braking only, a combination of mechanical and regenerative braking, and regenerative braking only. The result of a case study show that energy savings of at least 28% are possible by including regenerative braking.

Keywords Energy efficiency · optimal train control · regenerative braking

1 Introduction

Railway undertakings (RUs) are taking various measures to decrease energy consumption. Energy-efficient train control is an effective mean to reduce operating energy costs and so a lot of research is devoted to this area. Most of

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this research is based on optimal control theory, and in particular Pontryagin's Maximum Principle (PMP) (Pontryagin et al, 1962) has been applied to derive the necessary optimality conditions that characterize the optimal control structure (Howlett, 1996; Khmel'nitsky, 2000; Liu and Golovitcher, 2003; Scheepmaker and Goverde, 2015). The optimal control consists of a sequence of the optimal driving regimes maximum acceleration (MA), cruising (CR), coasting (CO), and maximum braking (MB). Given this knowledge of the optimal driving regimes, most train control algorithms then aim at finding the optimal switching points between the regimes.

However, most literature did not consider the possibility for regenerative braking, but only mechanical braking in which braking energy is transformed into heat. In regenerative braking, the released kinetic energy, generated by the engine of the train during braking (using electro-dynamic brakes), is sent back to the catenary system to be used by other surrounding trains. More and more trains nowadays have the ability to apply regenerative braking and therefore its impact on the optimal control strategy becomes highly relevant.

Asnis et al (1985) were the first to incorporate regenerative braking in the energy-efficient train control problem. They included regenerative braking in the objective function as a percentage of the total braking force and derived the necessary conditions for the optimal control strategy. In addition to the four basic optimal driving regimes, they found a fifth optimal driving regime of regenerative braking (RB). However, they did not provide an algorithm to find the optimal sequence and switching times of the driving regimes. Khmel'nitsky (2000) also considered the train control problem with regenerative braking. He found the same optimal control structure as Asnis et al (1985) and described a numerical method to solve the problem including varying gradients and speed limits. In an example he showed that regenerative braking was used to stabilize the cruising speed on downward slopes. Franke et al (2000) simplified the train resistance function and derived analytical expressions for the various driving regimes. They solved the problem by discrete dynamic programming and applied it to a case study. However, they did not provide details on the optimal sequence of driving regimes, but only mentioned that a main part of the computed energy saving resulted from the reduced use of the mechanical brakes. Baranov et al (2011) considered the train control problem considering both mechanical and regenerative braking in their objective function, and also included the efficiency of regenerative braking. They derived seven driving regimes: the three familiar regimes MA, CO and MB (maximum braking with both mechanical and regenerative braking), three cruising regimes with either partial traction (CR), partial regenerative braking (RB) or with full regenerative braking and partial mechanical braking, and finally a regime with full regenerative braking. An algorithm to construct the optimal control sequence of these driving regimes was mentioned as an open question. Rodrigo et al (2013) considered a discretized optimal train control problem including efficiency of regenerative braking and they considered both mechanical and regenerative braking for a metro system. They concluded from their metro simulations that regenerative braking from higher speeds is more efficient than

coasting, so they replace coasting by regenerative braking. Qu et al (2014) also considered metro systems and assume fully regenerative braking (no mechanical braking). They assume that the optimal driving strategy consist only of maximum acceleration, cruising and maximum regenerative braking. They provide an algorithm to calculate the optimal cruising speed and applied it to a case study of a metro line, but without reporting any energy savings.

In summary, the literature reports different impacts of regenerative braking on the energy-efficient train control depending on the assumptions and conditions. Therefore, this paper considers the energy-efficient train control problem with both mechanical and regenerative braking including efficiency and considers a case study for a main line local passenger train. In this paper we assume level track to focus on the structure of the optimal driving strategy without the influence of gradients.

Section 2 gives the energy-efficient train control problem without and with regenerative braking. Section 3 explains the algorithm applied to solve the train control problem, after which the model is applied on a case study of the Netherlands Railways (NS) in Section 4. Finally, Section 5 ends the paper with conclusions.

2 Model description

This section describes the energy-efficient train control (EETC) problem and derives the structure of the optimal driving regimes.

Consider the problem of driving a train between two stops in a given time with minimal energy consumption. Let X be the stop distance, T the scheduled running time. We use distance x as the independent variable and time $t(x)$ and speed $v(x)$ as function of position as the state variables. The control variable is the mass-specific force $u(x)$. The control variable can be partitioned into a nonnegative traction force and a negative braking force which are indicated as $u^+ = \max(u, 0)$ and $u^- = \min(u, 0)$, respectively. Note that implicitly $u^+ \cdot u^- = 0$, i.e., if u^+ is nonzero than $u^- = 0$ and vice versa, corresponding to the fact that trains do not give traction and brake at the same time.

Without regenerative braking, the energy-efficient train control problem can be formulated as (Scheepmaker and Goverde, 2015)

$$J = \min_u \int_0^X u^+(x) dx \quad (1)$$

subject to

$$t'(x) = 1/v \quad (2)$$

$$v'(x) = (u - r(v))/v \quad (3)$$

$$t(0) = 0, t(X) = T, v(0) = 0, v(X) = 0 \quad (4)$$

$$v(x) \in [0, v_{\max}], u(x) \in [-u_{\min}, u_{\max}(v(x))]. \quad (5)$$

The specific traction and braking force $u(x)$ are the forces acting on the train divided by total mass, including a rotating mass factor. The maximum mass-specific traction is a non-increasing function of speed. An example of the traction control-speed diagram with both a constant and hyperbolic part can be found in figure 1. The maximum braking is assumed to be constant. The mass-specific resistance force $r(v) = r_0 + r_1v + r_2v^2$ is a positive quadratic function of speed (Davis, 1926).

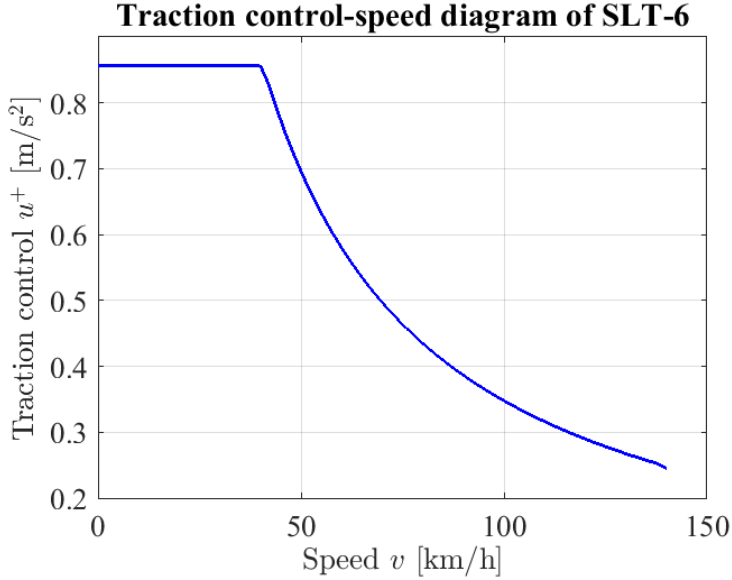


Fig. 1 Traction control-speed diagram for rolling stock type SLT-6 of NS

This problem can be solved by using optimal control theory. Pontryagin's Maximum Principle (PMP) can be applied to derive necessary conditions for the optimal control (Pontryagin et al, 1962). For this define the Hamiltonian

$$\begin{aligned}
 H(t, v, \varphi, \lambda, u) &= -u^+ + \frac{\phi}{v} + \frac{\lambda(u - r(v))}{v} \\
 &= \begin{cases} \left(\frac{\lambda}{v} - 1\right)u + \frac{\varphi}{v} - \frac{\lambda r(v)}{v} & \text{if } u \geq 0 \\ \frac{\lambda}{v}u + \frac{\varphi}{v} - \frac{\lambda r(v)}{v} & \text{if } u < 0, \end{cases} \quad (6)
 \end{aligned}$$

where $\varphi(x)$ and $\lambda(x)$ are the co-state variables satisfying the adjoint differential equations

$$\varphi'(x) = -\frac{\partial H}{\partial t} = 0 \quad (7)$$

$$\lambda'(x) = -\frac{\partial H}{\partial v} = \frac{\lambda u + \lambda v r'(v) - \lambda r(v) + \varphi}{v^2}. \quad (8)$$

The optimal control $\hat{u}(x)$ maximizes the Hamiltonian and is therefore obtained as

$$\hat{u}(x) = \begin{cases} u_{\max}(v(x)) & \text{if } \lambda(x) > v(x) & \text{(MA)} \\ u \in [0, u_{\max}] & \text{if } \lambda(x) = v(x) & \text{(CR)} \\ 0 & \text{if } 0 < \lambda(x) < v(x) & \text{(CO)} \\ -u_{\min} & \text{if } \lambda(x) < 0 & \text{(MB)}. \end{cases} \quad (9)$$

Clearly, the maximum control $\hat{u} = u_{\max}$ implies maximum acceleration (MA), zero control $\hat{u} = 0$ implies coasting (CO), i.e., rolling with the engine turned off, and the minimum control $\hat{u} = -u_{\min}$ implies maximum braking (MB). The singular solution defined by $\lambda(t) = v(t)$ corresponds to cruising (CR), i.e., driving at a constant optimal cruising speed using partial tractive effort $\hat{u} \in [0, u_{\max}]$. The actual optimal control is a sequence of these driving regimes, where the challenge is to determine the switching points between driving regimes. The optimal speed-distance profile for a level track is illustrated in Figure 2, with switching points x_1 , x_2 and x_3 .

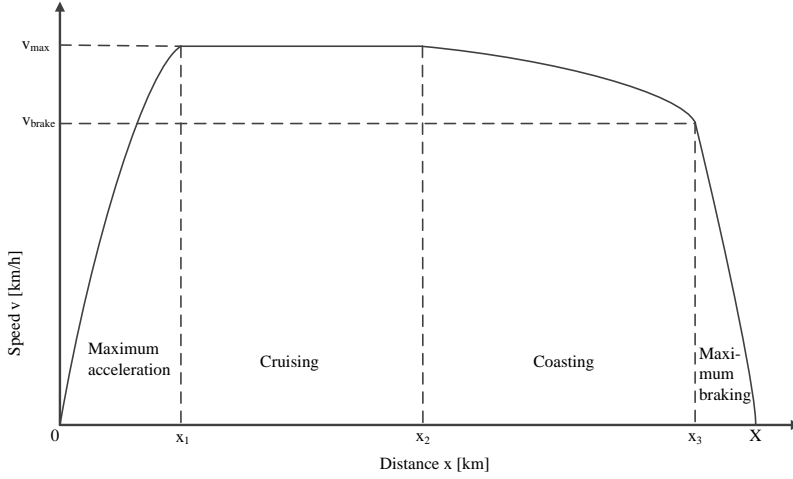


Fig. 2 Speed profile of a basic energy-efficient driving strategy on a level track with switching points between driving regimes at x_1 , x_2 and x_3

Regenerative braking is included into the model by partitioning the braking force into regenerative braking u_r and mechanical braking u_b , with $u^- = u_r + u_b$ and extending the objective function similar to Asnis et al (1985), Khmelnsky (2000) and Baranov et al (2011). The resulting optimal control problem can then be formulated as

$$J = \min_u \int_0^X (u^+(x) - \eta u_r(x)) dx, \quad (10)$$

subject to

$$t'(x) = 1/v \quad (11)$$

$$v'(x) = (u^+ - u_r - u_b - r(v))/v \quad (12)$$

$$t(0) = 0, t(X) = T, v(0) = 0, v(X) = 0 \quad (13)$$

$$v(x) \in [0, v_{\max}] \quad (14)$$

$$u^+(x) \in [0, u_{\max}(v(x))], u_r \in [0, \bar{u}_r], u_b \in [0, \bar{u}_b] \quad (15)$$

where $\eta \in [0, 1]$ is the efficiency of the regenerative braking system, \bar{u}_r is the maximum regenerative braking, and \bar{u}_b is the maximum mechanical braking with $\bar{u}_r + \bar{u}_b = u_{\min}$.

The resulting optimal control structure is now less obvious, since now there are three control variables with two braking controls. This leads to several options for braking by a combination of regenerative and mechanical braking, with the most likely according to Baranov et al (2011) being partial regenerative braking, maximum regenerative braking, maximum regenerative braking with partial mechanical braking, and maximum braking with both full mechanical and regenerative braking. The next section describes a direct algorithm to find the optimal control strategy without requiring a priori the structure of the optimal regimes.

3 Direct solution method

Numerical methods for solving optimal control problems are divided into two major classes: indirect methods and direct methods (Betts, 2010). In an indirect method, first necessary optimality conditions are derived based on e.g. Pontryagin's Maximum Principle. Then a solution is found satisfying the necessary optimality conditions which thus indirectly solved the original problem. This approach has been the preferred method in the train control community, where the original optimal control problem is translated into a finite optimization problem of finding the optimal sequence of optimal driving regimes and their switching points.

In a direct method, the state and/or control variables of the optimal control problem are discretized and the problem is transcribed to a nonlinear programming problem (NLP) (Betts, 2010). The NLP is then solved using well-known nonlinear optimization techniques. In particular, Pseudospectral methods have been proven themselves in solving optimal control problems efficiently (Rao, 2003). In Pseudospectral methods the control and state variables are parameterized by polynomials with exact values at the so-called collocation points. We applied the Gauss Pseudospectral Method (GPM) that uses orthogonal collocation at Legendre-Gauss points. For more details, see Rao (2003). In the train control community direct methods have been used only recently. Wang et al (2013) solved an optimal train control problem by transcribing it to both a GPM and a Mixed Integer Linear Programming Problem, and Wang et al (2015) used GPM to solve an optimal train control problem.

An efficient implementation of GPM is provided in the MATLAB toolbox GPOPS (General Pseudospectral OPTimal Control Software) (Rao et al, 2010a). We used Radau Pseudospectral method in GPOPS together with the automatic differentiator INTerval LABoratory (INTLAB) (Rao et al, 2010b; Rump, 1999).

4 Case study and results

The model called *EZ3R model* is tested in a case study. This case study and the results are discussed in this section.

The model is applied between the stations Driebergen-Zeist (Db) and Maarn (Mrn) in the Netherlands, see figure 3. Only the direction of Db to Mrn is considered in this paper. A local train of the Netherlands Railways (NS) (train series 7400) is running on this section stopping at both stations. The following data is available:

- Total distance between two stations is 7668 m.
- Total available running time according to the timetable of 2015 is 6 minutes.
- Maximum allowed speed at the section is 140 km/h.
- Rolling stock type SLT-6 (*Sprinter Light Train*) of NS is running on this section.



Fig. 3 Case study of local train series 7400 between Driebergen-Zeist and Maarn (inside black dotted square)

Besides, the following assumptions are made for modelling the problem:

- Maximum service braking (brake step 4 of SLT) instead of maximum braking is assumed for the the train, since this is more comfortable for the passengers and the train conductor in the train.
- No varying gradient profile and curves are used in the model.
- Signalling and ATP (Automatic Train Protection) system are not taken into account.
- Only the total traction energy consumption at the pantograph of a single train is considered, so no transmission losses of the energy (like regenerated energy) are taken into account and no other surrounding trains are taken into account.

The first step taken is to compare the model results generated by the *EZ3R model* with the *EZR model*. This is done by comparing both the time-optimal (TO) or technical minimum running time (i.e. a running a train as fast as possible from one station to the next station) and the energy-efficient train control (EETC) of both models. The *EZR model* is already calibrated, validated and verified and is in accordance with the results from optimal control theory (Scheepmaker and Goverde, 2015). Therefore, if the *EZ3R model* results are in line with the results of the *EZR model*, the *EZ3R model* is assumed to be plausible for further application.

The first important result of the model is that the calculation speed of the *EZ3R model* (5 s) is much faster than the the *EZR model* (122 s) (computed at a laptop with a 2.1 GHz processor and 8 GB RAM)).

Secondly, the model results are analyzed, which are visualized in figure 4. The results show that in general the *EZ3R model* generates results which are in line with the *EZR model*. However, the main differences exists in the energy consumption. For TO the *EZ3R model* calculates 0.7% higher energy consumption than the *EZR model*. For the EETC this is even higher, i.e. 2.5%.

Based on this analysis, it is concluded that the *EZ3R model* is a good alternative to the *EZR model* and the model results are comparable. Therefore, the *EZ3R model* is extended by including regenerative braking (RB), in order to investigate how RB will change the energy-efficient driving strategy. Three different models are compared based on the rolling stock data of NS:

1. EETC with only MeB (mechanical braking).
2. EETC with both MeB and regenerative braking (RB) (normal situation for SLT trains of NS).
3. EETC with only RB.

For the first two variants of the model, the total maximum braking force is the same (285.8 kN). This is because the electro-dynamic brakes can also be applied if the train cannot apply the regenerated energy (burn the regenerated energy as heat like mechanical braking). For the third variant, the total maximum braking force is less than the other variants (150 kN), since now the mechanical brakes are not applied.

The model results in GPOPS are generated within 20 s for the different braking variants. The results are displayed in figure 5. From the graphs the

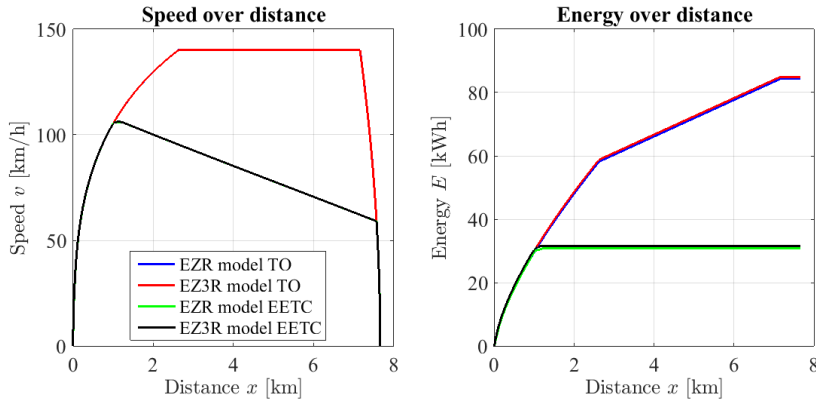


Fig. 4 Comparison of the time-optimal (TO) and energy-efficient train control (EETC) driving strategies between the *EZ3R model* and the *EZR model* (left speed-distance graph and right energy-distance graph)

effect of bang-bang control in GPOPS becomes clear for the driving strategies including RB. The fluctuation is clearly visible during the cruising driving regime. The traction control shows a lot of fluctuating peaks during this phase, which indicates that GPOPS has difficulties in calculating this bang-bang control during cruising.

Moreover, the main differences between EETC with MeB or RB become clear in the figure. If only MeB is applied, the train is accelerating to a higher speed than when RB is (also) applied. Secondly when RB is included, a cruising phase with a lower maximum speed is applied compared to MB only. The speed at which braking starts with RB is also higher than with only MeB, since RB now generates energy. It can be stated that the more energy that can be generated by RB, the higher the speed at the beginning of the braking phase will be. However, the maximum speed will always be lower than with MeB only, since a cruising phase with a lower maximum speed is included with RB.

The differences between EETC with MeB & RB and only RB are minimum. The energy consumption of only RB is slightly higher (1%) which can be explained by the fact that the available braking force with only RB is lower than combining both MeB and RB together. This means the train with only RB starts braking earlier and has a slightly higher cruising speed compared to the braking strategy with MeB & RB.

The results indicate that energy savings of 29.5% can be achieved by combining MeB and RB in the EETC compared to EETC with only MeB. The energy savings with only RB are a slightly lower, i.e. 28.8%.

So it can be concluded that including regenerative braking in EETC has a big influence on the extra decrease in total traction energy consumption. The effects of varying gradients (and varying speed limits) might even have a bigger effect on the total traction energy consumption for EETC with RB, since during downwards slopes RB can be applied to regenerate energy and absorb the potential energy. This will be a topic for future research.

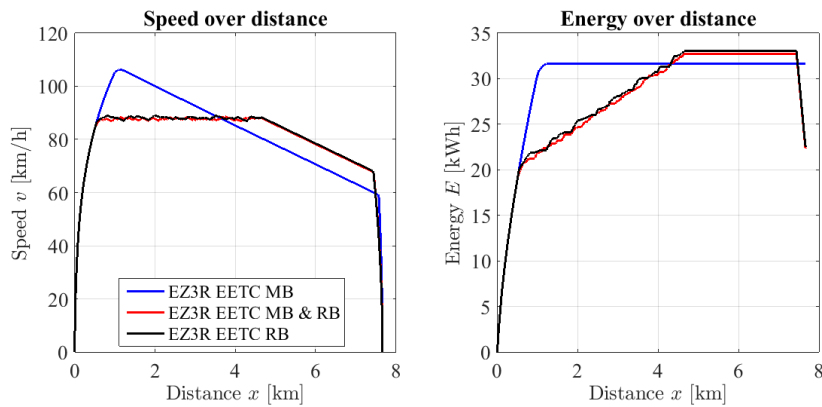


Fig. 5 Comparison between the energy-efficient train control (EETC) driving strategies with only mechanical braking (MeB), both MeB and regenerative braking (RB) or only RB of the *EZ3R* model (left speed-distance graph and right energy-distance graph)

5 Conclusion

In this paper an optimal train control problem has been considered in which total traction energy is minimized by including the effects of regenerative braking (RB). To compare the different energy-efficient train control (EETC) driving strategies with mechanical braking (MeB) and/or RB, an optimal control problem has been formulated and solved by the Gauss Pseudospectral Method.

The model has been applied in a case study in three scenarios: no regenerative braking, both regenerative and mechanical braking and only regenerative braking. The regenerative braking on level track had an impact on the optimal driving strategy. With regenerative braking the optimal cruising speed is lower than without, the coasting regime is shorter, and the braking regime starts earlier. This led to an extra energy saving of at least 28%. Based on this research the main conclusion is that regenerative braking does influence the energy-efficient driving strategy and leads to lower energy consumption.

In future research the model will be extended to include varying gradients and speed limits, as well as signalling constraints.

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