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Optimal Balancing of the Power Consumption of Trains in a Railway Network via Timetabling

Andreas Bärmann · Alexander Martin · Oskar Schneider

Abstract We investigate the problem of designing energy-efficient timetables for railway traffic. More precisely, we adapt a given base timetable before it is published by moderately shifting the departure times of the trains at the stations. To this end, we propose a mixed-integer programming model for feasible adaptations of the base timetable that possesses a rich assignment structure, which is very advantageous for the design of efficient solution procedures. We investigate its behaviour under different objective functions which fall into two classes: reducing the energy cost and increasing the stability of the power supply system. These tests are performed on real-world problem instances from our industry partner Deutsche Bahn AG. They show a significant potential for improvements in the existing railway timetables.

Keywords Energy-Efficiency · Mixed-Integer Programming · Power Load Balancing · Railway Traffic · Timetabling

Mathematics Subject Classification (2000) 90B06 · 90C11 · 90C27 · 90C57 · 90C90

1 Introduction

Railway traffic is the biggest individual electricity consumer in Germany. Its consumption amounts to 11 billion kilowatt-hours per year, which is 2 % of the total

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electricity usage in Germany or the consumption of the city of Berlin alone. The annual electricity bill of our industry partner Deutsche Bahn AG thus lies in the order of 1 billion Euros.

These figures indicate a high potential for cost savings by a more efficient use of electric energy. Deutsche Bahn AG already undertakes tremendous efforts to decrease the amount of energy consumed. But not only the total amount of energy used contributes to the energy cost. Up to 25 % of the electricity costs are due to the power load distribution, i.e. the temporal distribution of the energy demand. Usual electricity contracts for big electricity consumers include a price component proportional to the highest average power drawn over any 15-minute interval within the accounting period. Thus, it is advisable both from the view of the train operator and the electricity provider to schedule the power consumption such that this maximal average power load is as small as possible. On the other hand, a typical feature of the power load in railway energy systems are high fluctuations in the collective power drawn by the trains in the network. Increases and drops by 300 MW within few minutes are not unusual. Reducing the degree of fluctuation could help to increase the stability of the power supply system and to reduce the costs of the electricity provider as it enables a more regular operation of the power plants.

Both aims can be achieved by suitable adaptations of a planned timetable before it is finally published. To reduce the power component of the energy cost, it is sensible to avoid too many simultaneous departures as power consumption is highest during acceleration. On the other hand, trains can feed back part of the energy drawn from the catenary when braking. Therefore, departures should be synchronized with arrivals to make use of the recuperated energy by braking trains. These two measures also help to avoid high power fluctuations.

In these considerations, it may not be forgotten that the primary aim in railway transportation is the satisfaction of the customer. Therefore it is of course necessary to keep the level of service for the passengers from the tentative timetable and to meet all security requirements.

There is a lot of literature available that treats the energy-efficient operation of railway traffic. Most papers focus on reducing the energy consumption of the trains, which is possible by energy-efficient driving strategies (for example Doan et al (2014), Wang et al (2014), Li et al (2012), Miyatake and Ko (2010) and Boschetti and Mariscotti (2014)), operational measures (for instance Ragunathan et al (2014), Hasegawa et al (2014) and Kimura and Miyatake (2014)) or timetabling (see Su et al (2013), Peña-Alcaraz et al (2011), Li and Lo (2014), Fournier et al (2012) and Gong et al (2014)), among others.

On the topic of our work, the optimal balancing of the power load, there are still quite few publications. Sansó and Girard (1997) introduce a mixed-integer optimization model for adjusting metro timetables to minimize instantaneous peak power consumption and propose a heuristic to solve it. Similar timetabling approaches are taken in Kim et al (2011), Kim et al (2010) and Albrecht (2010).

An extensive review on different approaches for energy-efficient railway operations is given in Feng et al (2013). Optimization of the power load distribution has also been investigated in other problem contexts such as industrial production (see e.g. Lorenz et al (2012) and Nelson et al (2013)). For general approaches in railway timetabling, we refer to Cacchiani and Toth (2012).

Our work on balancing the power load induced by railway traffic is most directly related to that in Sansó and Girard (1997). After explaining the technical background in Section 2, we introduce a similar mixed-integer programming formulation for the adaptation of a tentative timetable by moderate shifts in the train departure times in Section 3. The main difference to the cited approach lies in the choice of the objective function. Instead of reducing instantaneous peak power consumption, i.e. the maximal power drawn at any single point of time by all trains together, we compare a variety of different objective functions to reduce their maximum average power consumption as well as fluctuations in power consumption. Our computational results Section 4 feature significant improvements in these two fields already for slight adaptations of a given timetable, which shows the viability of our models to enable reductions in energy cost and higher supply stability. The paper is rounded off with the conclusions and outlook given in Section 5.

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2 Technical Background

The intention of our models is the adaptation of a tentative railway timetable with respect to the temporal distribution of the power consumption by the trains. In the following, we briefly explain the functioning of a railway power supply system and discuss the importance of a well-balanced power load under economic and operational aspects.

Trains in an electric railway system are powered via catenaries or an electric rail, which in turn receive their power from a nearby supply facility, which is called a *substation*. The task of such a substation is to transform the electric current from a high-voltage long-distance power supply network to the voltage required by the engines of the traction units of the trains. In Germany, DB Energie GmbH, a subsidiary of Deutsche Bahn AG, maintains its own (almost) nationwide power supply network into which dedicated power plants directly feed their power. All individual power consumptions of the trains in the network accumulate on this level. Via various connections to the public power supply network, global under- or oversupplies can be compensated.

DB Energie GmbH charges the railway companies using the power supply according to two different quantities. Firstly, they have to pay for the total energy used by all vehicles over the course of the billing period, which typically comprises one month or a full year. This part of the electricity bill accounts for about 75 % of the total cost. Modern trains are able to recuperate part of the energy and to feed it back to the power supply, for which railway companies are refunded. As indicated above, a variety of approaches has been developed to reduce the total energy consumed by railway operations. However, there is a second major cost component making up for the other 25 % that depends on the temporal distribution of the energy consumption. The electricity meter within a traction unit not only measures the total consumption but also the instantaneous power drawn from the system, usually in a resolution of 1 second. This instantaneous power is summed over all trains in operation by the railway company and averaged over 15-minute intervals. The highest total consumption within any such interval in the billing period gives rise to the power component of the bill. This offers a great potential for cost savings that remains largely unused so far. Besides phases when a train climbs an inclination, it draws most power while accelerating. As a consequence, many simultaneous departures lead to costly peaks in consumption that could be avoided via desynchronization. From the view of the electricity provider, it would furthermore be lucrative to synchronize braking trains with accelerating trains in order to make better use of the recuperated energy. This would help to decrease its power dependent energy costs in a similar fashion as for the railway companies when drawing energy from the public supply network in times of power shortages.

This is where our models come into play. We assume to be given a preliminary timetable by the operator of the railway network, in Germany DB Netz AG, with some degree of freedom in the departure times of the trains from the stations. We try to exploit it by adjusting the planned departure times within small intervals of time, usually few minutes, in order to come to a more regular distribution of the overall consumption of the trains. This approach can be used with two different aims in mind. Firstly, we can shift the departures of the trains to achieve a system-wide power load minimizing the maximal average consumption over all 15-minute intervals of the planning horizon. For the electricity provider, these averages include the recuperated energy from braking trains. For the railway companies, recuperated energy is taken out of the averages as it is refunded seperately. This would be the optimal way to go from the cost point of view. From the view of the electricity provider, there is a second beneficial consideration. As already decribed, there are high fluctuations in power consumption within short time during normal operation of the railway network. Adaptation of the departure times may also be used to reduce these fluctuations to enable more regular generation schedules for the power plants supplying the system. This is another opportunity for cost savings as switching power plants on and off in a high frequency to adjust to the demand situation decreases the efficiency of their operation. To assess the potential for cost savings and to compare these two ways of looking at the problem is the aim persued in this paper. Figure 1 exemplarily shows the result of a timetable optimization as it is proposed here.

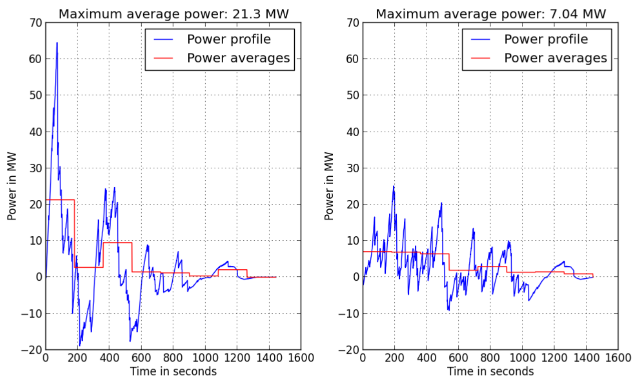


Fig. 1: Power load induced by railway traffic before (left) and after (right) optimization of the underlying timetable

On the left picture, we see the power load of the railway traffic induced by the underlying timetable before optimization. The power load experiences drastic jumps within very short intervals of time. Furthermore, there is a significant energy loss as there are long periods with highly negative power values. This energy recuperated from braking trains does not find other trains which could make efficient use of it. Altogether, this leads to very unfavourable, unbalanced power averages (visualized as a step function), which are a determining factor in the calculation of the energy cost. On the right picture instead, we see a much more balanced power profile where train departures are better desynchronised and where braking energy is more efficiently used. This leads to an advantageous average power profile whose maximum value could be reduced by a factor of three.

For our power load optimization models presented in the following section, we take several assumptions concerning the feasible adaptation of a preliminary timetable. The most important one is that the running times of the trains between two stations is not affected. That means we only adjust the dwell times of the trains at the stations, not their driving profile. Consequently, we assume the power consumption profile of each train for a trip between two consecutive stations given as input. We remark that this assumption may be relaxed by providing different possibilities for the driving profile of each trip. Furthermore, we assume that the train sequence on each track is given as input. That means that the order of departure of any two trains leaving a station via the same track needs to be preserved in the adjusted timetable. This leads to a very favourable assignment structure of the arising models, which allows for the solution of the problem for real-world size input data. In fact, the determination of optimal train sequences is part of the research of our partners in project E-Motion from TU Chemnitz (see Fischer and Helmberg (2014)). Finally, we require that all interchange relations between different trains in the stations are retained to ensure the same degree of service with respect to the passenger connections. The constraints making up a feasible timetable are explained in more detail in the next section where we derive a mathematical formulation of the problem.

3 Model Formulation

The timetabling problem established in the previous section is now formulated as a mathematical optimization problem. It takes the form of a mixed-integer program for which we define four different objective functions, leading to four different problem variants. We begin with the description of the feasible set, i.e. the set of all feasible timetables, which have to respect the technical constraints established above.

3.1 Feasible Timetables

Let the planning horizon of the optimization problem be given by the set $T = \{0, 1, 2, \dots, \Delta\}$, where Δ is its length in seconds. The railway network shall be given as a directed graph $G = (V, A)$ with the set V of stations and the set A of links connecting them. The set of trains to be scheduled will be denoted by R. Each single train $r \in R$ is defined by several parameters describing its journey.

Firstly, there is an ordered list V^r of serviced stations from V, excluding the final station, which is denoted by D^r. Secondly, there is another ordered list A^r which describes the corresponding links travelled between consecutive stations along the whole journey of the train. For each station $v \in V^r$ there is a set $J_v^r \subseteq T$ containing the feasible departure times for the train at this station. In our case, this will be all full minutes within a certain interval around the departure time of the tentative timetable. Furthermore, the travel time of the train on link $a \in A^r$ shall be given by Γ_a^r, while its minimum waiting time at station $v \in V^r$ is given by c_v^r, both values in seconds. The power consumption of train r on its trip along link a at t seconds after its departure, $0 \leq t \leq \Gamma_a^r$, is denoted by p_a^{rt}. For the formulation of the safety constraints, we also need a set L_a of all ordered pairs of consecutive trains on the same track of a link $a \in A$. Parameter $s_a^{r_1,r_2}$ then stands for the minimum headway time between the pair of consecutive trains $(r_1, r_2) \in L_a$ on that link. Recall that we assume the order of the trains on each link as given. For the interchange constraints we finally need a set U_v for each station $v \in V$ which contains all ordered pairs of trains $(r_1, r_2) \in R^2$ such that interchange from r_1 to r_2 remains possible after adapting the timetable. We demand that the departure of train r_2 must be at least ρ_v seconds later than the arrival of train r_1, but at most θ_v seconds later.

To model the set of feasible timetables, we introduce variables x_v^{rj} which take a value of 1 if departure of train $r \in R$ from station $v \in V^r$ takes place at second $j \in J_v^r$, and 0 otherwise.

A feasible timetable is then defined by the following constraints. Firstly and foremost, each train $r \in R$ has to depart from all stations $v \in V^r$ and its departure time has to lie within the prescribed set of possible departure times J_v^r. This requirement is stated as the following assignment constraint:

$$\sum_{j \in J_v^r} x_v^{rj} = 1 \quad (\forall r \in R)(\forall v \in V^r)(\forall j \in J_v^r). \tag{1}$$

Secondly, the scheduled departures of each train have to respect the minimum waiting times at the intermediate stations of its journey. This is formulated as a precedence constraint:

$$x_v^{rj} \leq \sum_{\substack{h \in J_w^r: \\ h \geq j + \Gamma_a^r + c_w^r}} x_w^{rh} \quad (\forall r \in R)(\forall a = (v,w) \in A^r, w \neq D^r)(\forall j \in J_v^r). \tag{2}$$

Then we have to enforce the minimum headway times between consecutive trains on the same track. This is done via two more precedence constraints – one for the departures and a second one for the arrivals. In the first case, we demand the second of two consecutive trains to wait at least the minimum headway time before it departs:

$$x_v^{r_1 j} \leq \sum_{\substack{h \in J_v^{r_2}: \\ h \geq j + s_a^{r_1 r_2}}} x_v^{r_2 h} \quad (\forall a = (v,w) \in A)(\forall (r_1, r_2) \in L_a)(\forall j \in J_v^r). \tag{3}$$

In the second case, we require it to arrive at least by the minimum headway time later at the subsequent station to prevent the two trains from getting to close to each other while passing the track:

$$x_v^{r_1 j} \leq \sum_{\substack{h \in J_v^{r_2}: \\ h \geq j + \Gamma_a^{r_1} - \Gamma_a^{r_2} + s_a^{r_1 r_2}}} x_v^{r_2 h} \quad (\forall a = (v,w) \in A)(\forall (r_1, r_2) \in L_a)(\forall j \in J_v^r). \tag{4}$$

The final constraint we consider concerns the interchange between the trains at the stations. For each pair of trains $(r_1, r_2) \in U_v$ at a station $v \in V$, we require that the arrival of r_1 and the departure of r_2 at this station lie within a time interval defined by ρ_v and θ_v in order to enable interchange from r_1 to r_2:

$$x_u^{r_1 j} \leq \sum_{\substack{h \in J_v^{r_2}: \\ h \geq j + \Gamma_a^{r_1} + \rho_v \wedge \\ h \leq j + \Gamma_a^{r_2} + \theta_v}} x_v^{r_2 h} \quad (\forall v \in V)(\forall (r_1, r_2) \in U_v, a = (u,v) \in A^{r_1})(\forall j \in J_u^{r_1}). \tag{5}$$

The above constraints for a feasible timetable are common to all four models defined in the following. We will refer to them as $x \in X$ for short.

3.2 Different Measures of the Balancedness of the Power Load

As indicated in Section 2, there are different thinkable measures of how well the power load arising from a given timetable is balanced, which depend on the intention of the planner. The four measures discussed in this paper – minimizing either the cost of the operator or the user of the power supply or minimizing the fluctuation of the power consumption according to two different norms – lead to four different objective functions to be optimized over the set of feasible timetables.

Before we introduce the resulting models, we define a common notation to assess the system-wide power load at a given instant $t \in T$. As we assume the power consumption profile for each trip of a train on a link as given, varying its departure time results in shifting the occurence of the corresponding power load forward or backward in time. Thus, if train $r \in R$ chooses departure $j \in J_v^r$ at station $v \in V^r$, its individual power consumption at instant $t \in T$ is given by

$$\bar{p}_a^{r,t-j} := \begin{cases} p_a^{r,t-j}, \text{ if } 0 \leq t - j \leq \Gamma_a^r \\ 0, \text{ otherwise} \end{cases}. \tag{6}$$

This allows us to calculate the system-wide power load at an instant $t \in T$ induced by some feasible timetable $x \in X$ via a function $P(x,t)$ defined as

$$P(x,t) := \max \left\{ \sum_{r \in R} \sum_{v \in V^r} \sum_{j \in J_v^r} \bar{p}_a^{r,t-j} x_v^{rj}, 0 \right\}. \tag{7}$$

It sums up all the individual power consumption profiles for each trip of the trains under consideration according to the chosen departure times. Note that energy recuperated by braking trains is lost if there are no others trains that would be ready to make use of it at the same time, thus the maximum with 0. Finally, we define $T_{15} := \{I \subset T \mid [900 \cdot i, 900 \cdot (i+1)] \cap T, i \in \mathbb{N}\}$ to be the set of all consecutive 15-minute (= 900-second) intervals within T (where the last interval may actually be somewhat shorter). For such a 15-minute interval $I \in T_{15}$, $t_0(I)$ and $t_{900}(I)$ shall denote the first and the last second of this interval respectively.

All the models presented in the following may easily be stated as mixed-integer linear programs (MIPs) by introducing auxiliary variables, if needed.

3.2.1 Reducing the Maximum Average Power – (TTMAP)

The first model considered here minimizes the highest average power consumption over all 15-minute intervals of the planning horizon. This is equivalent to minimizing the maximal energy consumed over all the 15-minute intervals. If we assume the power profile of a train r on a link a to be a piecewise-linear function, the energy-consumption over such an interval I is given by

$$E(x, I) := \frac{1}{2}(P(x, t_0(I) + P(x, t_{900}(I)) + \sum_{\substack{t \in I: \\ t \neq t_0(I) \wedge \\ t \neq t_{900}(I)}} P(x, t). \tag{8}$$

This term calculates the area under the system-wide power profile over interval I, which is the system-wide energy consumption over that interval.

To minimize the highest energy consumption over all 15-minute intervals, we have to solve the following optimization problem:

$$\min_{x \in X} \max_{I \in T_{15}} E(x, I). \tag{9}$$

It tries to find a TimeTable with a minimal Maximum Average Power consumption. We denote it by (TTMAP) for short. To obtain the resulting highest average power consumption, which determines the cost of the electricity provider, we have to devide its optimal value by 900.

3.2.2 Reducing the Maximum Average Pover before Recuperation – (TTMAPBR)

Our second model also minimizes the highest average power consumption over all 15-minute intervals. However, this time we do this for the *positive power profile* of each trip, i.e. the maximum of the original power profile with zero. The corresponding *gross power* drawn at instant $t \in T$ is then given by

$$P^+(x, t) := \sum_{r \in R} \sum_{v \in V^r} \sum_{j \in J_v^r} \max\{\bar{p}_a^{r, t-j}, 0\} x_v^{rj}. \tag{10}$$

Thus, our objective is to minimize the energy consumption *before recuperation* given by

$$E^+(x, I) := \frac{1}{2}(P^+(x, t_0(I) + P^+(x, t_{900}(I)) + \sum_{\substack{t \in I: \\ t \neq t_0(I) \wedge \\ t \neq t_{900}(I)}} P^+(x, t). \tag{11}$$

Note that in the above term, the calculation of the area under $P(x, t)$ over an interval I interchanges with the summation over the scheduled trips of the trains. Therefore, this objective function may be drastically simplified to

$$E^+(x, I) := \sum_{r \in R} \sum_{v \in V^r} \sum_{j \in J_v^r} E^+(r, v, j, I) x_v^{rj}, \tag{12}$$

where $E^+(r, v, j, I)$ is the energy consumption of such a trip before recuperation that falls into time interval I – a term that may be computed beforehand. As a consequence, the resulting optimization problem

$$\min_{x \in X} \max_{I \in T_{15}} E^+(x, I). \tag{13}$$

containes $|T|$ times fewer variables and constraints than (TTMAP) after linearization, which is quite considerable as the linearization leads to very dense rows. We denote this model by (TTMAPBR). To obtain the highest average power consumption before recuperation, the quantity determining the cost of the train operator, we have to divide its optimal value by 900 again.

3.3 Reducing the Power Fluctuation – (TTPF$_1$) and (TTPF$_\infty$)

Our last two models minimize the power fluctuation over the planning horizon to stabilize the power supply system. Generally, the power fluctuation is defined as the distance between the considered system-wide power profile and an ideally balanced, i.e. constant, power profile. As this distance can be measured in different norms, this approach leads to a family of optimization problems

$$\min_{x \in X} \sum_{t \in T} \|P(x, t) - m\|_p\,, \tag{14}$$

where m is an auxiliary variable that represents a mean power value according to some p-norm. From this family, we consider the two cases that lead to MIPs, namely $p = 1$ and $p = \infty$, where in the latter case, the problem may be simplified to

$$\min_{x \in X} \left(\max_{t \in T} P(x, t) - \min_{t \in T} P(x, t) \right), \tag{15}$$

i.e. the auxiliary variable may be dropped. We denote the two optimization problems by (TTPF$_1$) and (TTPF$_\infty$) respectively. While the first problem minimizes the summed deviation of the power consumption from the mean value over the planning horizon, the second problem minimizes the difference between the maximal and the minimal power consumption occurring during this time.

4 Computational Results

We have implemented all four models from the previous section using the Python-API of the commercial MIP solver Gurobi 5.6.3 (see Gurobi Optimization, Inc. (2014)). In the following, we present a comparative study of their computational complexity and their effects on the power load of the timetable under consideration. The instances considered here are real-world problem instances by our industry partner Deutsche Bahn AG. All computations have been performed on a compute server with Six-Core AMD Opteron$^{\text{TM}}$ 2435 2.6 GHz processores and 64 GB RAM. Each instance was solved using 4 cores and a time limit of 1 hour.

Before we present our results, we give a short description of the instances used for the study.

4.1 The Instances

All the instances in our benchmark set have originated from a data set provided by Deutsche Bahn AG that describes the Germany-wide railway timetable for passenger traffic for the year 2015. It includes all regional and long-distance passenger trains operated by DB Mobility Logistics AG, which represent about 80 % of railway passenger traffic in Germany.

The test instances for our timetable optimization problems represent the traffic during a time interval of 4 hours, from 8 a.m. until 12 noon, on a typical working day. Each instance has been created by choosing one railway station of a German city (a smaller or a bigger one) and including all trains (regional and long-distance traffic) which stop at this station during the time interval under consideration. In total, our test set consists of 18 instances. Their sizes range from 13 trains in the smallest instances up to 277 trains in the largest instance. For all our test instances, we provided Gurobi with the tentative timetable as an initial solution. Each departure time of a train at a station was then allowed to deviate by ± 3 minutes from the departure time established in the tentative timetable. As we only allowed the trains to depart at full minutes, this means that there are 7 possible departures for each trip.

The power profiles have been created according to the physical laws determining the power consumption of a train, using typical train parameters. We consider the acceleration power (based on plausible velocity profiles), the power required to overcome the downhill force (based on height data of the stations) the power required to overcome the rolling and the air friction as well as a minor random component. A typical power profile as it arises here can be seen in Figure 2.

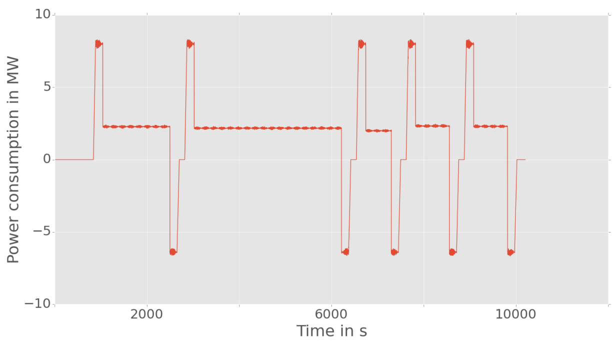


Fig. 2: The power profile of an ICE train on a journey with 5 trips

It shows the power consumption of an ICE on a journey with 5 trips along a quite constant inclination. Peaks in consumption are reached within the acceleration phases, while part of the energy can be recuperated when braking. Its consumption is limited by the engine power, which is 8 MW in the case of an ICE-3.

4.2 The Results

In the following, we present the results of all four models on the instances of our test set. Table 1 compares the outcome for the two models optimizing maximum averages peaks, (TTMAP) and (TTMAPBR).

Table 1: Results for Models (TTMAP) and (TTMAPBR) – For each instance, we state the solution time of the respective model or the resulting gap after 1 h as well as the improvement in the objective function.

	(TTMAP)		(TTMAPBR)	
Instance (#Trains, #Trips)	Time/Gap	Reduction	Time/Gap	Reduction
Zeil (13, 206)	45.92 %	13.8 %	9 s	18.7 %
Passau Hbf (26, 241)	0.17 %	36.5 %	47 s	14.4 %
Bayreuth Hbf (27, 77)	10.75 %	29.7 %	0 s	24.4 %
Jena Paradies (29, 244)	809 s	21.6 %	4 s	13.0 %
Lichtenfels (37, 352)	0.22 %	26.4 %	1 s	16.3 %
Erlangen (48, 676)	0.95 %	24.6 %	4 s	16.4 %
Bamberg (71, 863)	29.59 %	0.0 %	14 s	16.2 %
Aschaffenburg Hbf (73, 712)	0.51 %	22.1 %	61 s	17.1 %
Kiel Hbf (84, 482)	4.13 %	22.0 %	35 s	14.0 %
Leipzig Hbf (tief) (104, 1553)	27.71 %	0.0 %	118 s	8.0 %
Würzburg Hbf (117, 927)	15.31 %	0.0 %	65 s	9.0 %
Dresden Hbf (126, 1524)	25.52 %	0.0 %	0.02 %	12.1 %
Ulm Hbf (141, 1252)	22.48 %	0.0 %	2839 s	12.4 %
Stuttgart Hbf (tief) (155, 2340)	∞	0.0 %	287 s	6.1 %
Berlin Hbf (S-Bahn) (182, 3709)	∞	0.0 %	3143 s	3.1 %
Hamburg-Altona(S) (189, 2731)	∞	0.0 %	785 s	7.4 %
Frankfurt(Main)Hbf (237, 1808)	14.15 %	0.0 %	115 s	11.6 %
Nürnberg Hbf (277, 2641)	∞	0.0 %	172 s	9.3 %

We see at first sight that the two models differ greatly in computational complexity. This is not suprising because (TTMAPBR) possesses much fewer constraints and variables, as discussed in Section 3.2.2. In other words, it is a much easier problem to minimize the power dependent cost of the train operator than that of the electricity provider. Model (TTMAPBR) could be solved to optimality or close to optimality for all the instances under consideration. We also see that it attained considerable reductions in the maximum average power consumption over 15-minute intervals. They range from 9 % up to almost 25 %, which tells us that there is significant potential for cost reduction on the side of the train operator. Model (TTMAP) could only be solved to optimality on one of the smaller instances and close to optimality on 4 more instances. On most bigger instances, we observe quite high optimality gaps or even the case that the root relaxation could not be solved within the time limit (indicated by "∞"). However, the potential for cost reductions seems to be even higher, which is most probably explained by the fact that (TTMAP) can make use of the recuperated energy to smoothen the curve of 15-minute averages. Whenever the optimality gap stays moderate, the reduction is between 21 and 37 %. The instances with more than 100 trains were too hard to find better solutions than the initial one.

In Table 2, we show the results of the two models minimizing the fluctuation of the power consumption, (TTPF_1) and (TTPF_∞).

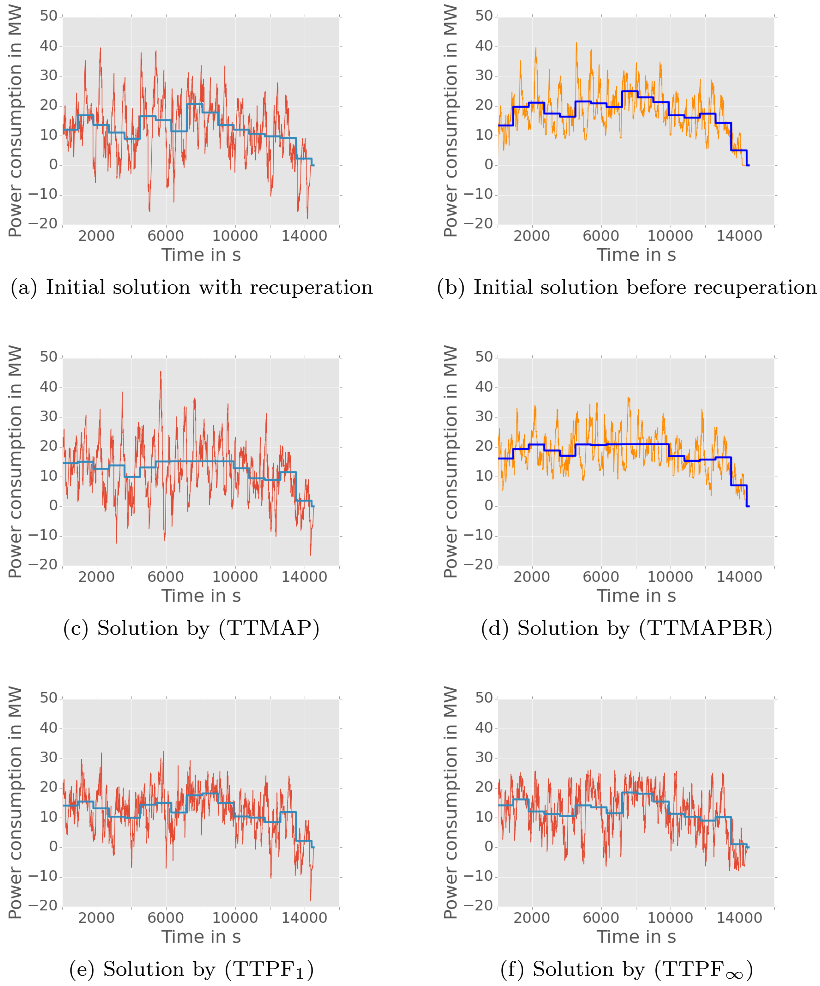
Table 2: Results for Models (TTPF_1) and (TTPF_∞) – For each instance, we state the solution time of the respective model or the resulting gap after 1 h as well as the improvement in the objective function.

	(TTPF_1)		(TTPF_∞)	
Instance (#Trains, #Trips)	Time/Gap	Reduction	Time/Gap	Reduction
Zeil (13, 206)	74.22 %	7.6 %	77.62 %	23.1 %
Passau Hbf (26, 241)	54.79 %	25.1 %	54.23 %	50.2 %
Bayreuth Hbf (27, 77)	45.27 %	27.7 %	30.08 %	55.9 %
Jena Paradies (29, 244)	41.68 %	24.3 %	21.85 %	48.1 %
Lichtenfels (37, 352)	36.26 %	26.4 %	33.91 %	40.8 %
Erlangen (48, 676)	61.07 %	0.0 %	49.81 %	20.8 %
Bamberg (71, 863)	65.81 %	1.7 %	40.28 %	41.7 %
Aschaffenburg Hbf (73, 712)	61.02 %	5.2 %	35.2 %	37.5 %
Kiel Hbf (84, 482)	56.05 %	2.6 %	33.15 %	36.9 %
Leipzig Hbf (tief) (104, 1553)	78.22 %	3.4 %	69.62 %	41.1 %
Würzburg Hbf (117, 927)	53.38 %	0.0 %	40.57 %	24.6 %
Dresden Hbf (126, 1524)	62.91 %	6.0 %	51.83 %	25.9 %
Ulm Hbf (141, 1252)	71.55 %	16.0%	∞	0.0 %
Stuttgart Hbf (tief) (155, 2340)	52.94 %	0.0 %	64.5 %	0.0 %
Berlin Hbf (S-Bahn) (182, 3709)	∞	0.0 %	56.67 %	0.0 %
Hamburg-Altona(S) (189, 2731)	∞	0.0 %	62.07 %	0.0 %
Frankfurt(Main)Hbf (237, 1808)	45.42 %	0.0 %	32.3 %	23.9 %
Nürnberg Hbf (277, 2641)	∞	0.0 %	28.44 %	38.3 %

Here, we observe high optimality gaps already for the small instances, which shows that these models are harder to solve than those minimizing the maximum average consumption. This can be explained by the fact that their linearization basically contains each power profile twice as linearizing an absolute value requires a lower and an upper bound. Model (TTPF_1) tends to be somewhat more difficult to solve than (TTPF_∞), which might be due to the additional auxiliary variable for the mean value around which power consumption fluctuates. Nevertheless, we see that both models are able to reduce these fluctuations visibly on many instances. Model (TTPF_∞) is able to reduce the margin of fluctuation by over 50 % in two of the cases. For the electricity provider this means that high peaks and low valleys in consumption are avoided which allows for a much more regular operation of the facilities satisfying the power demand.

4.3 One Instance in Detail

The comparative study of our four models on the 18 test instances showed that considerable reductions in either the maximum average power or the power fluctuation are possible. We now show the effects of the optimization of the system-wide power profile at the example of instance *Lichtenfels*. We choose this somewhat smaller instance, as all models are able to achieve significant reductions in their respective objective functions, which allows to characterize the differences between the solutions more easily. The series of pictures given in Figure 3 shows the the system-wide power profile as the summation of the power profiles of the individual trips before and after optimization. They also depict the arising 15-minute averages over the planning horizon.



(a) Initial solution with recuperation

(b) Initial solution before recuperation

(c) Solution by (TTMAP)

(d) Solution by (TTMAPBR)

(e) Solution by (TTPF_1)

(f) Solution by (TTPF_∞)

Fig. 3: Comparison of the system-wide power profile before and after optimization of test instance *Lichtenfels* under the different models considered here

Figure 3a shows the initial solution in the case taking into account recuperated braking energy. The optimized solution resulting from Model (TTMAP) can be seen in Figure 3c. We observe that the power profiles before and after optimization do not differ too much from each other, as they still show high peaks and low valleys in consumption. The maximal consumption has even increased from 40 to 46 MW. However, the maximal 15-minute average is significantly lower after the optimization – a decrease from 21 to 16 MW. For the electricity provider, this means a decrease in the power-dependent cost by about 25 %. It becomes visible that this effect was achieved by smoothening the step function of 15-minute averages by balancing the load over the 15-minute intervals of the planning horizon. The same findings apply to Figures 3b and 3d, which show the situation with-out recuperation. Note that in these two pictures, the positive power profiles are shown, and the averages are calculated with respect to these curves. The maximal average value drops from 25 to 21 MW, which means a cost reduction on the side of the train operator by 16 %.

Now we turn our attention to reducing the power fluctuation. Therefore, we compare the initial solution in Figure 3a with Figures 3e and 3f, which show the solutions by Models (TTPF_1) and (TTPF_∞) respectively. The effect on the system-wide power profile is quite easy to see. Both models try to avoid high peaks and low valleys by synchronizing acceleration phases with braking phases as well as possible. The solution of (TTPF_∞) is able to reduce the margin of fluctuation from almost 60 MW to about 35 MW – a reduction by 40 %. This bandwith is of course higher for optimization according to the 1-norm by Model (TTPF_1). But in return, it is able to avoid much fluctuation within the resulting bandwith. Note that the resulting curve of 15-minute averages is quite unsmooth in both cases. This is a strong indication that balancing these averages and reducing the fluctuation in consumption are very different aims of optimization. Which one of the two is better suited thus strongly depends on the point of view onto the problem.

5 Conclusions and Outlook

We have presented four models to optimize the adaptation of a tentative railway timetable with respect to different objective functions concerning the efficiency of the power load of the trains. We were able to show that their solutions enable considerable cost savings both for the electricity provider and the companies operating the trains in real-world problem instances. Our future work will be dedicated to developing algorithms to treat larger instances. The aim here is the optimization of the Germany-wide timetable. Furthermore, we need to take possible delays in the daily operations into account which could quickly render the effect of adapting a timetable useless. This calls for the incorporation of robust optimization techniques into our models.

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