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Template Based Re-Optimization of Rolling Stock Rotations

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Abstract Rolling stock, i.e., rail vehicles, are among the most expensive and limited assets of a railway company. They must be used efficiently applying optimization techniques. One important aspect is *re-optimization*, which is the topic that we consider in this paper. We propose a *template concept* that allows to compute cost minimal rolling stock rotations under a large variety of re-optimization requirements. Two examples, involving a *connection template* and a *rotation template*, are discussed. An implementation within the rolling stock rotation optimizer ROTOR and computational results for scenarios provided by DB Fernverkehr AG, one of the leading railway operators in Europe, are presented.

Keywords Combinatorial optimization, mixed integer linear programming, railway scheduling

1 Introduction

Rolling stock, i.e., rail vehicles, are the most expensive and limited assets of a railway company and must therefore be used efficiently. The *Rolling Stock Rotation Problem* (RSRP) addresses this task. It deals with the cost minimal construction of rolling stock rotations to operate a given timetable of passenger trips by rail vehicles, including a large number of operational requirements like vehicle composition rules, maintenance constraints, infrastructure capacity constraints, and regularity requirements. A detailed problem description, a

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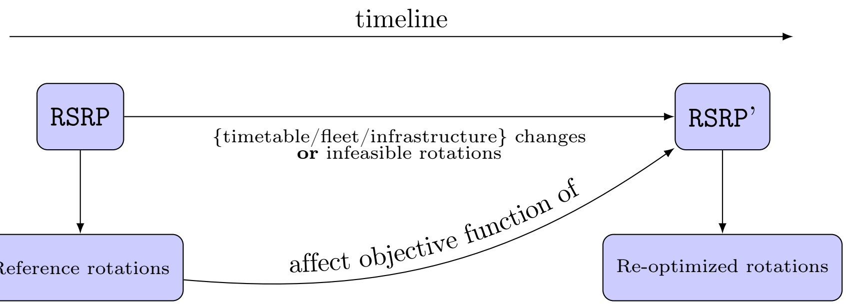


Fig. 1 Concept of Re-optimization for the RSRP.

mixed integer programming formulation, and an algorithm to solve this problem in an integrated manner is described in detail in Reuther et al (2012b).

In this paper we focus on the *re-optimization* task arising in rolling stock planning, see Reuther et al (2014). The concept can be summarized as follows. At some point in time a railway undertaking has to tackle an instance of the RSRP and constructs a rotation plan. At another point in time conditions and assumptions of the problem change, such that the existing reference rotation plan can no longer be operated. The reasons for that can be manifold, e.g.,:

- driver lockouts as well as strikes,
- exceptional weather conditions,
- holidays implying timetable changes,
- planned construction sites,
- unpredictable accidents or technical failures,
- and many more.

In such situations, a new problem RSRP′ has to be solved. The most important difference to greenfield planning is that there exist rolling stock rotations, namely the *reference rotations*, that were already implemented in operation. Crew was scheduled for vehicle operations and maintenance tasks, capacity consumption of parking areas was reserved, and most important in a segregated railway system, e.g., in Europe and Germany: train paths were already allocated for the deadhead trips. Additionally, in big railway companies the rolling stock is subdivided into smaller sets of railway vehicles which operate the different rotations. This is done for economical and operational reasons. Hence, a major goal in constructing a solution to RSRP′ is to change as little as possible in comparison to the reference rotation plan. Several column generation based linear programming approaches to tackle these kinds of problems have been proposed in the literature, see in Haahr et al (2014); Huisman (2007); Budai et al (2010).

2 The re-optimization model

In this section we provide an overview on the hypergraph based model proposed in Reuther et al (2012a) with a particular focus on the main modeling ideas. For technical details including the treatment of maintenance and capacity constraints see Reuther et al (2012a). The extension of the following problem description and model to include maintenance constraints is straight forward and does not affect the content or the contribution of the paper. Nevertheless, in our computational study we provide results for instances with maintenance constraints.

2.1 The rolling stock rotation problem

We consider a cyclic planning horizon of one *standard week*. The set of timetabled passenger trips is denoted by T. Let V be a set of *nodes* representing departures and arrivals of vehicles operating passenger trips of T, let $A \subseteq V \times V$ be a set of directed standard arcs, and $H \subseteq 2^A$ a set of *hyperarcs*. Thus, a hyperarc $h \in H$ is a set of standard arcs. The RSRP *hypergraph* is denoted by $G = (V, A, H)$. The hyperarc $h \in H$ *covers* $t \in T$, if each standard arc $a \in h$ represents an arc between the departure and arrival of t. We define the set of all hyperarcs that cover $t \in T$ by $H(t) \subseteq H$. By defining hyperarcs appropriately vehicle composition rules and regularity aspects can be directly handled by our model.

The RSRP is to find a cost minimal set of hyperarcs $H_0 \subseteq H$ such that each timetabled trip $t \in T$ is covered by exactly one hyperarc $h \in H_0$ and $\bigcup_{h \in H_0} a$ is a set of *rotations*, *i.e.,* a set packing of cycles (each node is covered at most one time).

We define sets of incoming and outgoing hyperarcs of $v \in V$ in the RSRP hypergraph G as $H(v)^{\text{in}} := \{h \in H \,|\, \exists\, a \in h \,:\, a = (u, v)\}$ and $H(v)^{\text{out}} := \{h \in H \,|\, \exists\, a \in h \,:\, a = (v, w)\}$, respectively. By using a binary decision variable for each hyperarc, the RSRP can be stated as a mixed integer program as follows:

$$\min \sum_{h \in H} \mathbf{c}_h x_h, \tag{MP}$$

$$\sum_{h \in H(t)} x_h = 1 \qquad \forall t \in T, \tag{1}$$

$$\sum_{h \in H(v)^{\text{in}}} x_h = \sum_{h \in H(v)^{\text{out}}} x_h \qquad \forall v \in V, \tag{2}$$

$$x_h \in \{0, 1\} \qquad \forall h \in H. \tag{3}$$

The objective function of model (MP) minimizes the total cost of operating a timetable. For each trip $t \in T$ the covering constraints (1) assign exactly one hyperarc of $H(t)$ to t. The equalities (2) are flow conservation constraints for each node $v \in V$ that imply the set of cycles in the arc set A. Finally, (3) state the integrality constraints for our decision variables.

2.2 Re-optimization

The major re-optimization requirement for the RSRP is to change as little as possible of the reference rotation plan. We argue that this requirement can be handled by defining a suitable objective function based on the reference rotation plan.

$$\mathbf{c} : H \mapsto \mathbb{Q}_+ \ : \ \mathbf{c}(h) := \left\langle \begin{pmatrix} \mathbf{c}_1(h) \\ \mathbf{c}_2(h) \\ \mathbf{c}_3(h) \\ \mathbf{c}_4(h) \\ \mathbf{c}_5(h) \\ \mathbf{c}_6(h) \\ \mathbf{c}_7(h) \\ \mathbf{c}_8(h) \end{pmatrix}, \begin{pmatrix} p_1(h) \\ p_2(h) \\ p_3(h) \\ p_4(h) \\ p_5(h) \\ p_6(h) \\ p_7(h) \\ p_8(h) \end{pmatrix} \right\rangle \begin{array}{l} \ldots \ \textit{connection deviations} \\ \ldots \ \textit{composition deviations} \\ \ldots \ \textit{service deviations} \\ \ldots \ \text{vehicles} \\ \ldots \ \text{services} \\ \ldots \ \text{deadhead distance} \\ \ldots \ \text{regularity} \\ \ldots \ \text{couplings} \end{array} \tag{4}$$

Definition (4) illustrates our approach. Our objective function is the sum of the re-optimization cost $\sum_{i=1}^{3} \mathbf{c}_i p_i$ and the original objective function $\sum_{i=4}^{8} \mathbf{c}_i p_i$. We propose to compute the parts of the re-optimization objective as a sum of costs depending on individual hyperarcs.

Let $h \in H$ be a hyperarc. In a first step we reinterpret h in the reference rotations, *i.e.,* we search the timetabled trips that are connected or covered by h in the reference rotation plan, if they still exist. The reinterpretation procedure is very precise as a node in our hypergraph has the following attributes w.r.t. the vehicle traversing the node: position in a composition, orientation w.r.t. driving direction, fleet type, and rotation (*i.e.,* cycle) of a vehicle.

In a second step we compute a property $p_i(h) \in \mathbb{N}$ for $i = 1, \ldots, 4$ for $h \in H$ that states the number of differences of h w.r.t. the reference rotations. Examples for such differences are:

- Let $h \in H$ a hyperarc connecting the timetabled trips t_1 and t_2. If t_1 and t_2 exist in the reference rotations and both trips are not connected there, we set $p_1(h) = |h|$. In all other cases we set $p_1(h) = 0$.
- If h covers trip t that exists in the reference rotations and is operated by a different vehicle composition than h, we set $p_2(h) \geq 1$, otherwise $p_2(h) = 0$. The exact numeric number depends on $|h|$, how these vehicles are oriented, which fleets are used etc..
- If h implies a different maintenance service before or after a timetabled trip $p_3(h) = 1$, otherwise $p_3(h) = 0$.

Solutions of re-optimization instances often have the characteristic that major parts of the reference rotations are not changed, but some small parts have to be modified. In some cases, however, new timetabled trips have to be incorporated in the reference rotation plans. To handle this case we also have to consider properties of the original objective function $\sum_{i=4}^{8} \mathbf{c}_i p_i$ for re-optimization instances, *i.e.,* costs for vehicles consumed by a hyperarc, costs for maintenance services, costs for deadhead distances, cost for irregularities, and costs for coupling activities. Finally all of these individual properties are multiplied by corresponding cost parameters $\mathbf{c}_i$, $i = 1, \ldots, 8$ that can be customized to the requirements of industrial use cases.

In this way we are able to handle a lot of technical re-optimization details simply by changing objective coefficients. This allows us to apply the general model and algorithm presented in Reuther et al (2012a) to solve re-optimization instances.

Nevertheless, the described approach is not able to reproduce rotations exactly in case where multiple rotations for the same railway vehicle types exist. Furthermore, real world re-optimization scenarios show that the recognition process of individual task is to strict. Therefore, we introduce two templates to overcome these limitations which are described in the next section.

3 The template approach

A *template concept* for the re-optimization of duty schedules is proposed in Borndörfer et al (2013). The main idea is to define *duty templates* which are able to model "regional"[1] requirements such as the (more or less detailed) distribution of breaks in duties. To this end, a pricing problem with a dedicated graph is solved for each duty type template individually. The individual graphs allow to model the regional requirements and can be seen as copies of some original graph. These copies have been modified in order to provide a template for some regional structures.

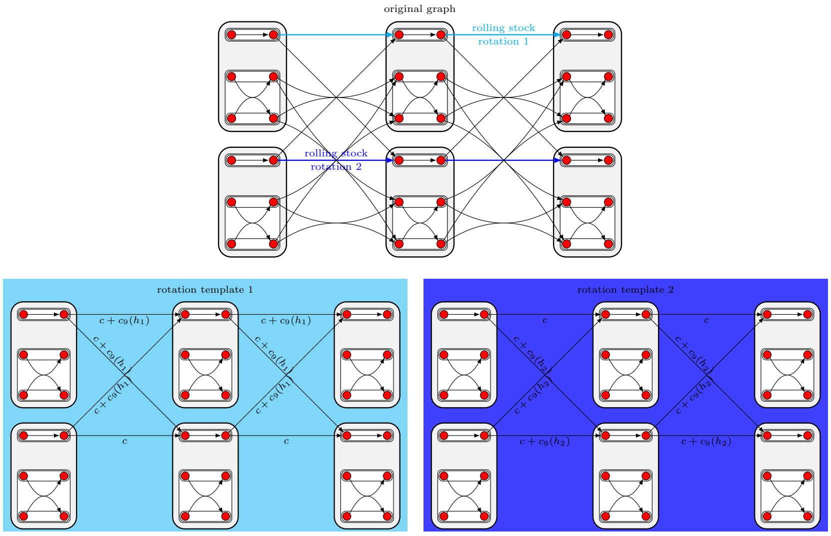
We follow this line in order to address the cyclic rolling stock re-optimization task. A more or less regional requirement for the re-optimization of rolling stock rotations is that connections between timetabled trips should be planned as similar as possible to the reference rotation plan – we introduce obvious *connection templates* for that purpose.

In fact, a more regional requirement appears by considering the *set* $T_r \subseteq T$ of timetabled trips which are operated by a dedicated rolling stock rotation r of the reference rotation plan. It is desired that "most of" the timetabled trips of $T_r \subseteq T$ still appear in the same rotation *after* re-optimization. We use *rotation templates* for that requisite.

Rotation templates and duty templates are somehow related in the sense, that a (partial) copy of some original graph is made for their implementations. Whereas we create a dedicated graph for "all duties with 2 breaks in the morning and 1 break after lunch" in duty scheduling, we create a dedicated

[1] "regional" fuzzily identifies some requirement that is not completely local and also not completely global, it is rather in between.

Fig. 2 Definition of a rotation template for two rolling stock rotations of the original problem



hypergraph for "all rotations that cover a set of timetabled trips". By introducing appropriate set-partitioning constraints both, the rotations for timetabled trips and the duties for duty elements are assigned in the re-optimization models.

Note that the modeling trick of coping something in order express a dedicated situation can be interpreted as a sampling technique which plays an important role in robust and stochastic optimization, see e.g., Ben-Tal et al (2009) and Eisenblätter and Schweiger (2012).

Obviously, this simple modeling trick might have disadvantageous effects on the sizes of the (hyper-) graphs as well as on the fractionality of corresponding solutions of linear programming relaxations. Indeed, it turns out that the fractionally is often reduced by considering a reference solution (in particular when large parts do not have to be changed, which is the usual use case in industry) and this increase in integrality is paying for the increase in size.

Our contributions are modeling gadgets, namely the *connection template* and the *rotation template*, for rolling stock rotations. This is done in order to increase the usability of our re-optimization model and to transfer the methodology of templates for re-optimization to rolling stock rotation planning.

3.1 Rotation templates

Let R be the set of rolling stock rotations that appear in the reference rotations and let $G = (V, A, H)$ a original hypergraph as introduced in Section 2.1. In order to re-optimize towards the rotations R, we introduce an extended hypergraph G_R:

$$G_R = \left(\bigcup_{r \in R} V_r, \bigcup_{r \in R} A_r, \bigcup_{r \in R} H_r \right).$$

The *rotation template* for $r \in R$ is the hypergraph $G_r = (V_r, A_r, H_r)$. G_R originates from G by creating a copy for each node, arc, and hyperarc of the hypergraph G for each rotation $r \in R$. Thus, rotation templates induce copies of almost (a copy can be truncated in some situations when, e.g., the fleets of the reference rotations mismatch with the feasible fleets for a trip) the whole original graph w.r.t. the reference rotations. This provides almost full control over the reconstruction of parts of the solution. Furthermore, it allows the direct penalization of most of all kinds of potential deviations, e.g., different connection, changed position within the composition, or a change of orientation.

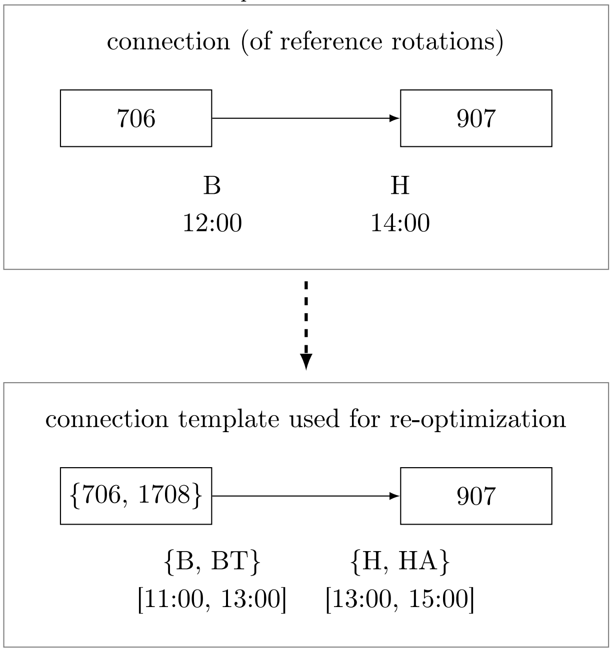
The construction of the objective coefficients of the rotation templates works as follows. Each of the "reference rotation dependent" hyperarcs $h_r \in H_r$ has an updated objective function coefficient which depends on r. For this we define a property $p_9(h_r)$ in order to penalize the deviation from the reference rotation r. If a trip t implied by hyperarc h_r is not operated in rotation r of the reference solution, we simply set $p_9(h_r) = |h_r|$ and zero otherwise. The updated cost coefficient reads $\mathbf{c}_{h_r} = \mathbf{c}(h_r) := \sum_{i=1}^{9} \mathbf{c}_i p_i(h_r)$ for h_r. The value of the parameter $\mathbf{c}_9$ is specified by the user to penalize the rotation deviations. The reference rotation dependent definition of the hypergraphs gives us complete control of the assignment of trips to different rotations and therein minimal deviations.

Figure 2 shows an example of two reference rolling stock rotations for the original graph (above) and two copies of the graph representing the corresponding rotation templates of the rolling stock rotations (below). Red circles mark arrivals or departures of trains. Boxes including two red circles symbolize the operation of a trip with the corresponding vehicle. Arcs that connect two different boxes show the possible preceding or succeeding trips. Hyperarcs mark operations in double traction. On the left hand side we introduce the copy of the hypergraph based rotation template of rolling stock rotation 1 and on the right hand side the copy for rolling rotation 2, respectively. In both cases it can be seen how deviating from the reference solution, i.e., the rolling stock rotation changes, can directly be penalized on the corresponding (hyper-)arc. In this case arcs have cost c if they fulfill the rotation template definition or $c + \mathbf{c}_9(h_r)$ in case of a violation. Furthermore, arcs representing a different vehicle assignment to the trips are not copied, see the hyperarcs of the original graph modeling the operation of the trips with double traction.

3.2 Connection templates

Connections between timetabled trips appear in the reference rotations. These connections have the following main characteristics:

Fig. 3 Definition of a connection template based on a connection of the reference rotations



– identifier of the arriving and departing trips,
– arrival and departure times, as well as
– arrival and departure locations.

The *connection templates* are motivated by the fact that in operational practice small changes of the trip characteristics, i.e., platform, position, orientation, or temporal changes are usual. To increase the robustness of the recognition process of trips from the reference rotation and the considered timetable, we introduce *connection templates* to provide regular operations. The *connection templates* are also used to group together regular operations which repeat on multiple days of the *standard week*. The definition of the *connection templates* consist of a temporal and a spatial part. Let W be the set of all connections of the reference rotations. To define the *connection templates* we define for all trips $t \in T$ a set of similar identifier I_t, e.g., all trip identifier for trips that share the same arrival an destination location. Further, we define for each location l a set of neighborhood locations N_l. Both sets could have cardinality one if there are no similar identifier or locations. Additionally, we define a deviation time window $\Delta_\tau := [\alpha, \beta]$ for each arrival or departure time τ. Let $t_{arr}(h), t_{dep}(h)$ be the trip identifier, $l_{arr}(h), l_{dep}(h)$ the locations, and $\tau_{arr}(h), \tau_{dep}(h)$ the times for arrival respectively departure of a hyperarc or connection.

In order to optimize towards W, we define as *connection template* for reference connection $w \in W$ the set of hyperarcs $H_w \subseteq \bigcup_{r \in R} H_r$ where for each

$h \in H_w$ holds that trip identifier of h are in the sets for similar identifier of w, the arrival and departure locations of h are in the neighborhood sets of w and the arrival, and departure times fit in the deviation time windows of w.

We set $p_9(h) = 0$ $h \in H_w$ if $r(h) = r(w)$.

A connection template covers a connection if its train identifier, arrival, and departure locations belong to the sets of its alternatives and its arrival and departure times fit in the given time windows.

Figure 3 shows an example of a connection template. The left hand side visualizes a connection for trips with identifier 706 and 907 arriving in B at 12:00 and departing in H at 14:00. On the right hand side a connection template for this connection is shown, accepting trip identifier 706 or 1708 as arrival and 907 as departure, arrivals at B or BT between 11:00 and 13:00 and departures in H or HA between 13:00 and 15:00 to match the template. For the practical applications of our industrial partner there are different use cases for the connection templates. They are used for minor changes of the start or final location of a trip or even if the itinerary of the trip changes, e.g., if a maintenance of the original itinerary takes place.

3.3 The template model

The modified *template model* which includes both types of templates reads as follows.

$$\min \sum_{r \in R} \sum_{h_r \in H_r} \mathbf{c}_{h_r} x_{h_r}, \tag{TMP}$$

$$\sum_{r \in R} \sum_{h_r \in H_r(t)} x_{h_r} = 1 \qquad \forall t \in T, \tag{5}$$

$$\sum_{h_r \in H(v_r)^{\text{in}}} x_{h_r} = \sum_{h_r \in H(v_r)^{\text{out}}} x_{h_r} \qquad \forall v_r \in V_r, r \in R \tag{6}$$

$$x_{h_r} \in \{0, 1\} \qquad \forall h_r \in H, r \in R \tag{7}$$

The objective function of model (TMP) minimizes the total cost of operating a timetable. For each trip $t \in T$ the covering constraints (5) assign exactly one hyperarc of $H(t)$ to t. The equalities (6) are flow conservation constraints for each node $v \in V$ that imply the set of cycles in the arc set A. Finally, (7) state the integrality constraints for our decision variables.

4 Computational study: The benefit of templates

In this section, we report on a set of computational experiments carried out to assess the effectiveness of the template approach implemented within the rolling stock optimizer ROTOR, see Reuther et al (2012b). Furthermore, we evaluate the impact of the different templates on the model's tractability and on the quality of produced solutions in practice. This implementation makes use of the commercial mixed integer programming solver `CPLEX 12.6` as internal LP solver. All our computations were performed on computers with an Intel(R) Xeon(R) CPU E31280 with 3.50 GHz, 8 MB cache, and 16 GB of RAM in multi thread mode with four cores.

Table 1 Key numbers of the instances

instance	trips	trip distance	compositions	fleets	maintenances	disturbed trips	recognizable trips	$\|V\|$	$\|H\|$
RSRP_1	788	555686	2	2	8	187	22	5702	10026804
RSRP_2	788	555686	2	2	8	189	22	5702	10027556
RSRP_3	665	265657	2	1	0	198	184	7680	16465044
RSRP_4	793	430770	11	5	32	50	7	8350	33113628
RSRP_5	785	426459	11	5	32	169	120	8330	33069736
RSRP_6	53	37501	5	3	20	40	40	498	138558
RSRP_7	670	263602	3	2	0	27	27	7650	16645161
RSRP_8	670	263602	3	2	0	27	25	7642	16596517

The main characteristics, i. e., distance and number of trips, number of compositions, number of fleets, and number of maintenances, are shown in Table 1. Columns marked with *disturbed* and *recognizable trips* give the number of trips in the timetable that deviate from those of the reference solution including additional trips and the number of those trips that are recognizable.

For each instance we ran four experiments for each combination of with or without rotation or connection template. Tables 2 shows the number of trips that deviate from their reference rotations, the total and the rotation dependent costs of the solution and the computation needed for the computation using the rotation templates. Tables 3 give the numbers for the computations with rotation templates. All computations finish with an gap of less than 1%. In case of computations without the rotation template the solutions were evaluated with the objective function of the rotation template computations.

Analysing Table 3 reveals that using connections templates is a first step towards producing similar rotations in comparison with the given reference solution. It reduces the number of unrecognised trips which leads to an increased number of penalized rotations that deviate from the reference rotations. This penalization has an significant influence on the run time of the computations which can be seen in both tables. The more sufficient step to preserve the reference rotations is indeed are the rotation templates. Its usage decreases the deviations of trips from their reference rotations significantly. Additionally, it has a positive effect on the computation time as it makes deviating rotation much more unattractive which can be seen in Table 3.

Table 2 Key numbers of the computational results with `ROTOR 2.4` and `CPLEX 12.6` without using rotation templates

	no connection templates				connection templates			
instance	rotation deviations	total costs	rotation dep. costs	computation time	rotation deviations	total costs	rotation dep. costs	computation time
RSRP_1	0	2372353	96248	14786	0	2364368	91248	9451
RSRP_2	0	2477205	197853	15653	0	2475174	189853	13396
RSRP_3	200	4146977	301069	5661	200	4145977	300069	4012
RSRP_4	15	5783234	148178	16299	15	5783234	148178	22105
RSRP_5	358	6308657	570314	37543	111	6001832	258314	19878
RSRP_6	0	497744	43323	10	0	476165	20323	51
RSRP_7	462	8565642	3534730	5338	455	8551786	3520730	4508
RSRP_8	484	8613483	3567730	3511	411	8528327	3482730	4091

Table 3 Key numbers of the computational results of `ROTOR 2.4` and `CPLEX 12.6` with using rotation templates

	no connection templates				connection templates			
instance	rotation deviations	total costs	rotation dep. costs	computation time	rotation deviations	total costs	rotation dep. costs	computation time
RSRP_1	0	2314496	42220	6034	0	2313562	40441	1698
RSRP_2	0	2456665	176853	5265	0	2331487	46166	1775
RSRP_3	0	3946977	101069	2052	0	3866027	28424	1806
RSRP_4	0	5745234	110178	6557	0	5666657	41572	6785
RSRP_5	0	5827728	158314	17172	0	5808472	64592	8650
RSRP_6	0	486744	25323	46	0	459663	4021	34
RSRP_7	7	7959391	2913730	4492	7	6444310	1892170	2306
RSRP_8	6	7983810	2913730	4076	6	6467000	1892290	2087

5 Conclusion

We have shown that the hypergraph model for the re-optimization case of the RSRP can be extended by the template concept. Moreover, it turns out that templates are a powerful concept that allows us to compute cost minimal rolling stock rotations under a large variety of requirements for re-optimization scenarios operated by DB Fernverkehr AG.

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