The Rapid Transit Frequency and Fleet Size Setting with Maximal Profit

Alicia De los Santos, **Juan A. Mesa,**
University of Seville
Federico Perea, Polytechnic University of Valencia

Outline

- Rapid Transit Systems
- Line Planning Objective Functions
- Data and notation
- Variables
- Objective function
- Constraints
- MIP Approach
- Solution procedures

Rapid Transit Systems (RTS)

- Metro, Underground, U-bahn
- Ligh Metro
- Light Rail
- Monorail
- Commuter train

Segregated from other modes though there are hybrid systems (German Stadtbahn, light rail, etc.)

Rapid Transit Systems (RTS)

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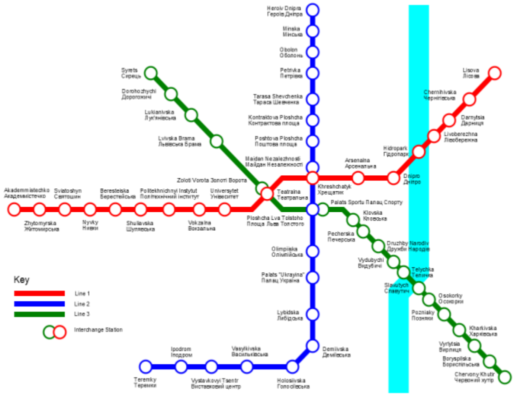


Rapid Transit Systems (RTS)

- Metro/Underground/U-bahn,etc.: 191 cities, 49 inaugurated in the 21st century. 36 in construction
- China: 22 metros in operation, 36 for 2020
- Commuter systems: ?
- Most of the new metros are in the “3rd world” in very big cities

Rapid Transit Systems (RTS)

Main features of RTS

- RTS have characteristics of both railway network and public transit
- Operate in cities and metropolitan areas
- Carry a large number of passengers traveling on short distances
- Headways are usually very short

Rapid Transit Systems (RTS)

- RTS compete (and sometime co-operate) with other modes of transportation (bus, car, bicycle)
- Often line railway tracks are not connected and platforms of different lines in multiple stations are at different levels
- In old systems (Paris, London, New York) some transfers need several minutes

Rapid Transit Systems (RTS)

- Regarding the sequential planning process:

1st Stage: Line network design (infrastructure design of lines)

2nd Stage: Frequency and capacity setting

- Since the demand is time-dependent and elastic the frequency and capacity setting problem has to be solved for different periods of the day, week, seasons, etc

Line Planning

- Bussiek, Optimal Line in Public Rail Transport (1997)
- Goessens, Van Hoesel, Kroon, A Branch-and-cut Approach for Solving Railway Line-planning Problems (2004)
- Schöbel, Line Planning in Public Transportation: Models and Methods (2012)
- Gallo, Montella, D’Acierno, The Transit Network Design Problem with Elastic Demand (2011)
- Schmidt, Integrating Routing Decision in Public Transportation Problems (2014)

Objective functions

Two groups

Oriented to customers: travelling and riding time, number of direct travelers, number of transfers

Oriented to the operator company: fixed and variable costs

Objective functions

- Relationship between costs and incomes
- PROFIT = REVENUE - COST
- It is not only a operator-oriented objective because revenue depends on ridership which is an efficiency indicator.

The problem

- Aim:

  To assign the frequency per hour, and the number of carriages, to each line, so that the net profit is maximized

Data and Notation

- **Let** $N = \{i_1, \dots, i_n\}$ **be the set of stations and** $\mathcal{L} = \{\ell_1, \dots, \ell_{|\mathcal{L}|}\}$ **the line set**
- **A line** $l \in \mathcal{L}$ **can be represented by a chain graph with edges** $\{i_{j_1}, i_{j_2}\}$
- **The line network is** $((N, E), \mathcal{L})$ **where** E **is the union of all edges of the lines**
- **Let** d_{ij} **be the length of edge** $\{i, j\}$ **and** λ **the commercial speed**

Data and Notation

- **Let $\nu_\ell = 2L_e/\lambda$ be the cycle time of ℓ , being the length L_e**
- **Let $G_{E'} = (N, E')$ be the network of the alternative mode, d'_{ij} being the length of edge $\{i,j\} \in E'$**
- **Let $W = \{w_1, \ldots, w_{|W|}\}$ be the set of OD pairs and g_w the demand of pair w**
- **u_w^{ALT} is the travel time of the pair w using the alternative mode**

Data and Notation

Cost structure

- **Let $c_{loc}\ and\ c_{carr}$ be the operating cost of the locomotive and that of one carriage per unit of length**
- **c_{crew} is the personnel cost per train and year**
- **$I_{loc}\ and\ I_{carr}$ are the adquisition cost of locomotive and carriage, respectively**

Capacity

- **⊖ is *the* capacity (the number of passengers) of a carriage**
- **$y^{\min}$ is the minimum number of carriages of a train**

Data and Notation

- η is the ticket fare
- τ is the passenger subsidy
- ρ is the number of hours per year
- $\widehat{\rho}$ is the time horizon in years
- Δ_i is the time spent in transferring at i
- $\mathcal{H} \subset \mathcal{Z}^+$ is the set of possible headways

Variables

- $x_l \in \mathcal{H}$ is the headway of line ℓ
- $y_l \in \mathcal{Z}^+$ is the number of carriages of trains of line ℓ
- u_w^{RTS} travel time of pair w using RTS
- p_w^{RTS} proportion of demand of w captured by the RTS
- $f_{ij}^{w\ell} = 1$ if ℓ traverses arc (i,j) using line ℓ, 0 otherwise
- $t_i^{w\ell\ell'} = 1$ if w transfers in staton i from line ℓ to line ℓ', 0 otherwise.
- $B_l \in \mathcal{Z}^+$ is the required fleet of line ℓ measured in number of trains

Variables

- Travel time of w in RTS

= waiting time + in-vehicle time + transfer time

$$\begin{aligned} u_w^{RTS} &= \sum_{\ell\in\mathcal{L}}\sum_{j:\{w_s,j\}\in\ell}\frac{x_\ell f_{w_s j}^{w\ell}}{2} + \frac{60}{\lambda}\sum_{\ell\in\mathcal{L}}\sum_{\{i,j\}\in\ell} f_{ij}^{w\ell} d_{ij} \\ &+ \sum_{\ell\in\mathcal{L}}\sum_{\ell':\ell'\neq\ell}\sum_{i\in\ell\cap\ell'} t_i^{w\ell\ell'}\left(\frac{x'_\ell}{2}+\Delta_i\right), \quad w=(w_s,w_t)\in W \end{aligned}$$

Variables

- Modal split

$$p_w^{RTS} = \frac{1}{1+e^{\alpha-\beta(u_w^{ALT}-u_w^{RTS})}}, \quad w \in W$$

- Required fleet as a function of the headway

$$B_l = \lceil 120 L_\ell / x_\ell \lambda \rceil$$

Objective Function

- Net Profit = Revenue - Cost

$$z = z_{rev} - (z_{rc} + z_{fic} + z_{cr})$$

$$\begin{aligned} z = \quad & \rho\hat{\rho}(\eta+\tau)\textstyle\sum_{w\in W} g_w p_w^{RTS} \\ & -\rho\hat{\rho}\textstyle\sum_{\ell\in\mathcal{L}} \lambda B_\ell (c_{loc} + y_l c_{carr}) \\ & -\textstyle\sum_{\ell\in\mathcal{L}} (I_{loc} + I_{carr} y_\ell) \\ & -\hat{\rho} c_{crew} \textstyle\sum_{\ell\in\mathcal{L}} B_\ell \end{aligned}$$

Constraints

- Transfer Contraints

$$t_k^{w\ell\ell'} \geq \sum_{j:(k,j)\in\ell} f_{kj}^{w\ell} + \sum_{i:(i,k)\in\ell'} f_{ik}^{w\ell`} - 1$$

$$w \in W, \ell \neq \ell' \in \mathcal{L}, k \in \ell \cap \ell', k \neq w_s, w_t$$

Constraints

- Flow conservation constraints

$$\sum_{\ell\in\mathcal{L}}\sum_{i:(i,k)\in\ell} f_{ik}^{w\ell} - \sum_{\ell\in\mathcal{L}}\sum_{j:(k,j)\in\ell} f_{kj}^{w\ell} = \begin{cases} 0, k \in N \setminus \{w_s, w_t\} \\ -1, k = w_s \\ +1, k = w_t \end{cases}$$

Constraints

- Upper bound on the number of passengers

$$x_\ell \sum_{w\in W} g_w p_w^{RTS} f_{ij}^{w\ell} \leq 60\Theta y_\ell, \quad \ell \in \mathcal{L}, \{i,j\} \in E$$

$$y_\ell \in \mathcal{Z}^+, \quad f_{ij}^{w\ell} \in \{0,1\}, \quad x_\ell \in \mathcal{H} \subset \mathcal{Z}^+$$

$$t_k^{w\ell\ell'} \in \{0,1\}, p_w^{RTS} \in [0,1]$$

- Thus it is a Mixed Integer Non-Linear Programming (MINLP) program

MIP approach

- The product $p_w^{RTS} f_{ij}^{k\ell}$ can be linearized
- The logit can be approximated by a piecewise linear function
- The required fleet uses the non-linear ceiling function and the headway is in the denominator. Considering the headway as a parameter the problem becomes a $ILP(x_1, \ldots, x_{|\mathcal{L}|})$

ILP-based Algorithm

- **Input Data**: Line Network, Demand (G), parameters
- **For** each combination of headways

$$(x_1, \ldots, x_{|\mathcal{L}|})$$

solve $ILP(x_1, \ldots, x_{|\mathcal{L}|})$

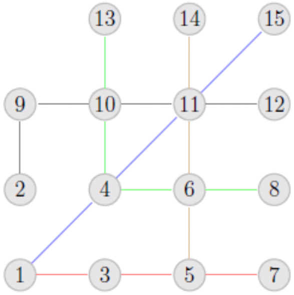
- **end**
- **Compute** $\arg\max_{(x_1,\ldots,x_{|\mathcal{L}|})} ILP(x_1, \ldots, x_{|\mathcal{L}|})$

Passenger-oriented Algorithm

```
Input Data: Line Network, Demand (G), Parameters
for each combination of headways do
  let Z={ }
  Compute the shortest path for each OD pair
  and the number of passengers traveling on
  each line and arc
  for each line  do
      Find the arc   with maximum load;
      Find the minimum number of carriages needed to
      transport all passengers traversing
  end  Compute the profit   $z_{NET}$   and keep this value $Z=Z\cup\{z_{NET}\}$
end
Compute the maximum net profit
Output: headways and capacities for maximum profit
```

An example

instance	profit (java)	nb trips (java)	CPU time (java)	profit (gams)	nb trips (gams)	CPU time (gams)
seed1	14653625433,00	42507,73804	25,524	14653625433,00	42507,73804	2906
seed2	14798133449,00	42769,27686	25,429	14798133449,00	42769,27686	2757,831
seed3	13967809433,00	40805,87791	25,49	13967809433,00	40805,87791	2793,178999
seed4	14934703086,00	43792,43812	25,467	14934703086,00	43792,43812	2758,832
seed5	13556226517,00	39396,90291	25,851	13556226517,00	39396,90291	2703,566
seed6	13269902666,00	40182,34353	25,129	13269902666,00	40182,34353	2740,568
seed7	14786292666,00	43306,02259	25,767	14786292666,00	43306,02259	2708,875
seed8	15134405421,00	44312,23282	25,445	15134405421,00	44312,23282	2792,209999
seed9	12626577094,00	38857,12863	25,503	12626577094,00	38857,12863	2804,843
seed10	15002705176,00	43824,09965	25,328	15002705176,00	43824,09965	2767,416



The lines are defined as: red line $\ell_1 = \{1, 3, 5, 7\}$, blue line $\ell_2 = \{1, 4, 11, 15\}$, green line $\ell_3 = \{13, 10, 4, 6, 8\}$, gray line $\ell_4 = \{2, 9, 10, 11, 12\}$ and brown line $\ell_5 = \{5, 6, 11, 14\}$.

Figure 1.9.: Representation of 15 × 5-configuration.

Further research: The Capacitated Problem

- CROWDING IN TRS
- “The unpleasant experience of too many passengers fitting into a confined space thus worsening passenger’s wellbeing”
- Several effects: platform crowding, excessive waiting time, increased dwell time, in-vehicle time

Crowding in TRS

- Platform crowding
- Excessive waiting time

Crowding in TRS

- Increased dwell time
- In-vehicle crowding

Crawding

The End

Thank you for your attention!!

LOGRO

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