An optimisation framework for determination of capacity in railway networks

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RobustRailS

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Agenda

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3) Capacity consumption in railway networks

4) Optimisation framework continued

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    2) Capacity results

6) Solution space

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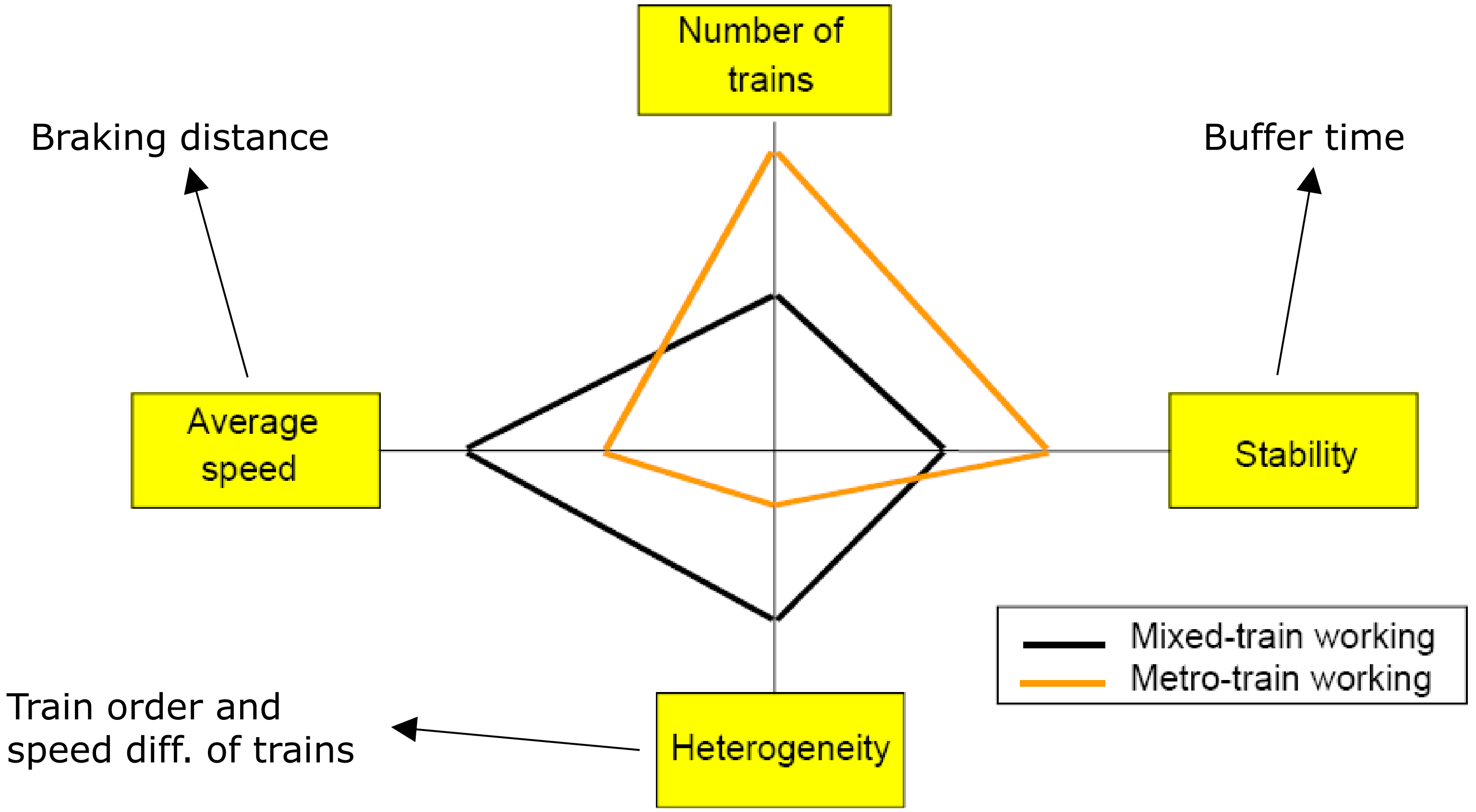
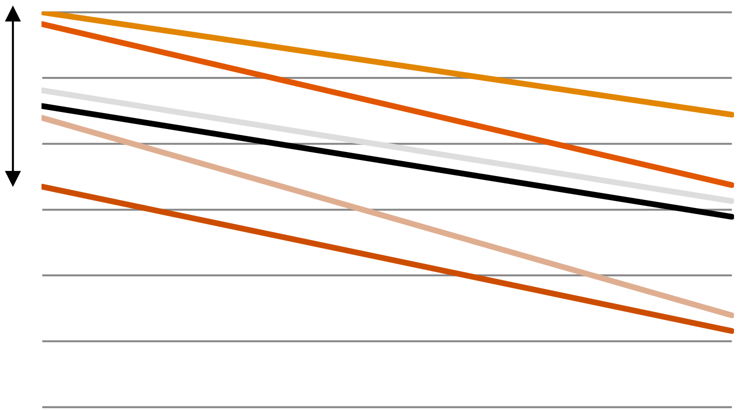
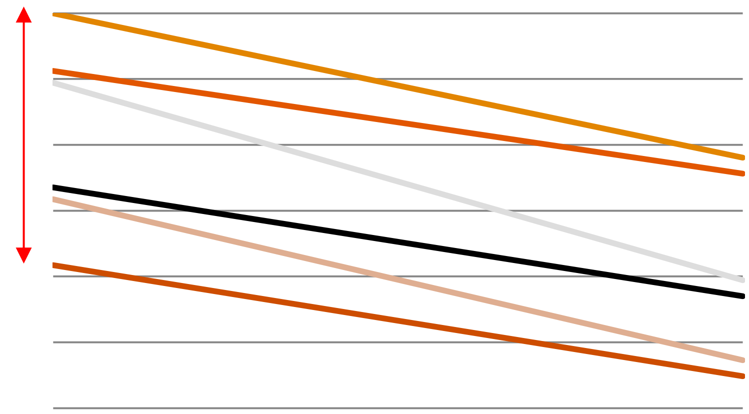
What is railway capacity?

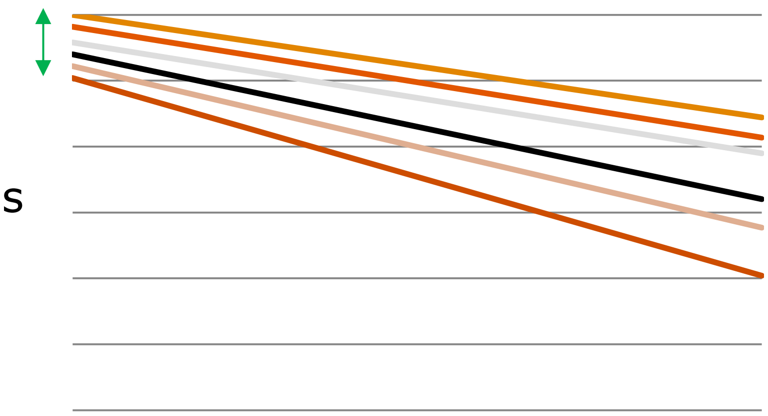
"Capacity as such does not exist. Railway infrastructure capacity depends on the way it is utilised." (UIC, 2004)



How to mix trains - Heterogeneity







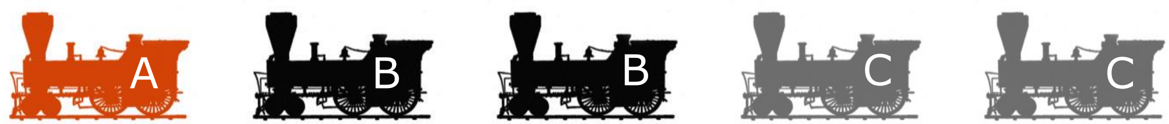
- n! for acyclic timetables
- (n-1)! permutations for cyclic timetables
- If more than 1 copy of a train type (k):
  - $n!/(k_1!k_2!\ldots k_m!)$
    (Multinomial coefficient)

Objective

- *What is the number of trains that a railway network can handle under a given robustness threshold?*
- Strategic planning: no train sequence/timetable is given beforehand
- As railway capacity depends on the sequence of trains,
  a span of number of trains must be found rather than a single value

Approach for determining railway capacity

- A train mix is given, e.g.
  - 20% of type A, 40% of type B and 40% of type C

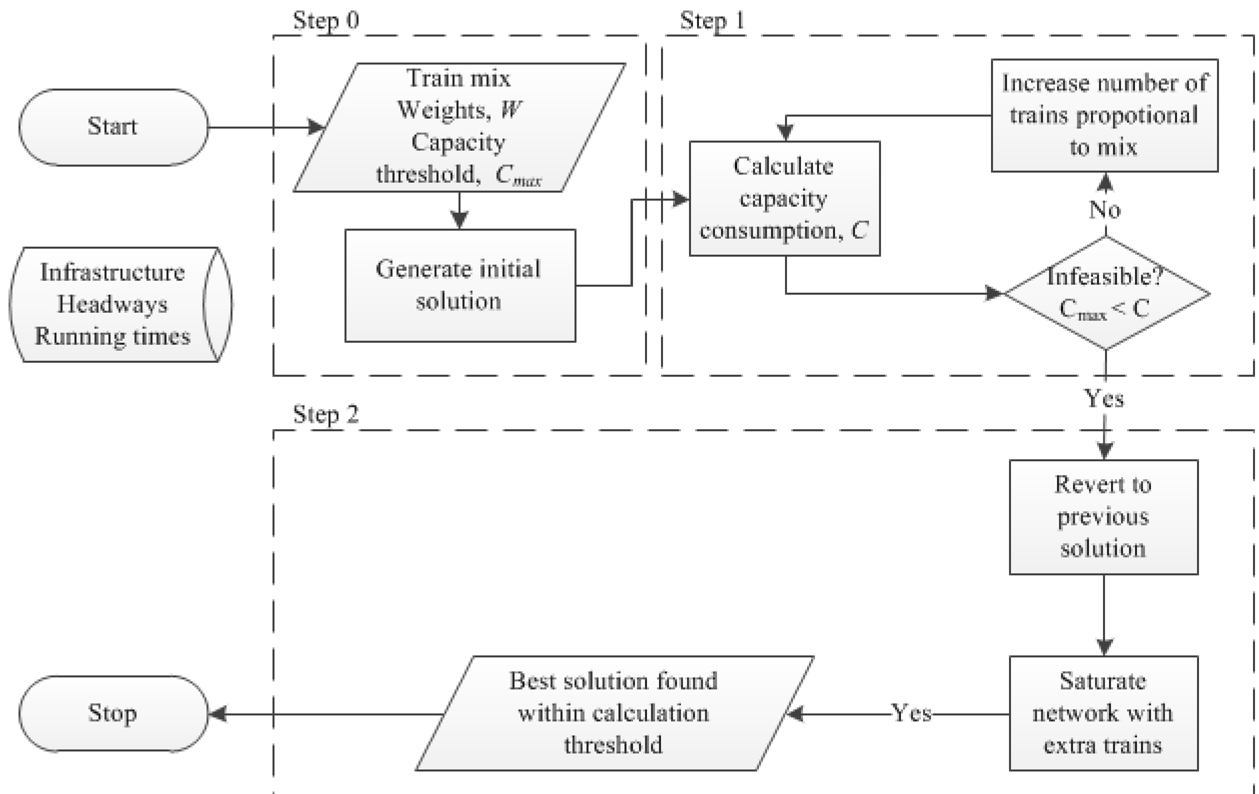


- The output is trains that can pass the network
  - According to mix, e.g.

  - Extra trains that saturate the network according to weights, e.g.

Framework

Optimisation framework:



Measuring capacity consumption

- The model/method used for capacity consumption measurement must fulfil the following requirements:
  - If an extra train is added to a network:

    The capacity consumed will never decrease compared to the previous solution (without the extra train)
  - Likewise, if a train is removed the capacity consumed will never increase

- If the span of capacity must be obtained, the model has to account for this as well (heterogeneity)

Our capacity consumption model

Input

- The network
  - Infrastructure
  - Headways
- Train type runs
  - Route
  - Running time
  - Number of each type

Calculate

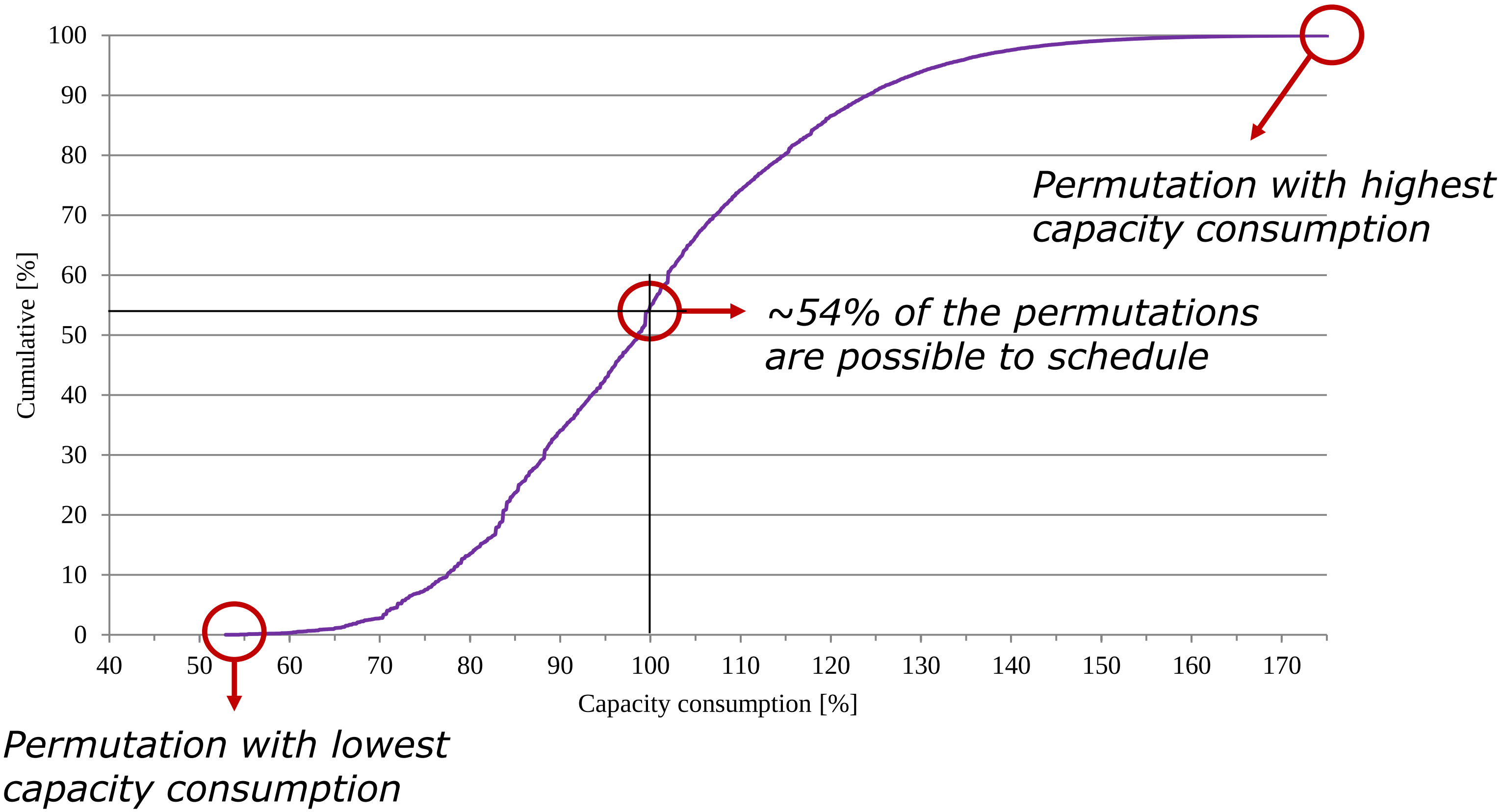
Output

- (Distribution of) capacity consumption

Calculate capacity consumption for each (random) permutation

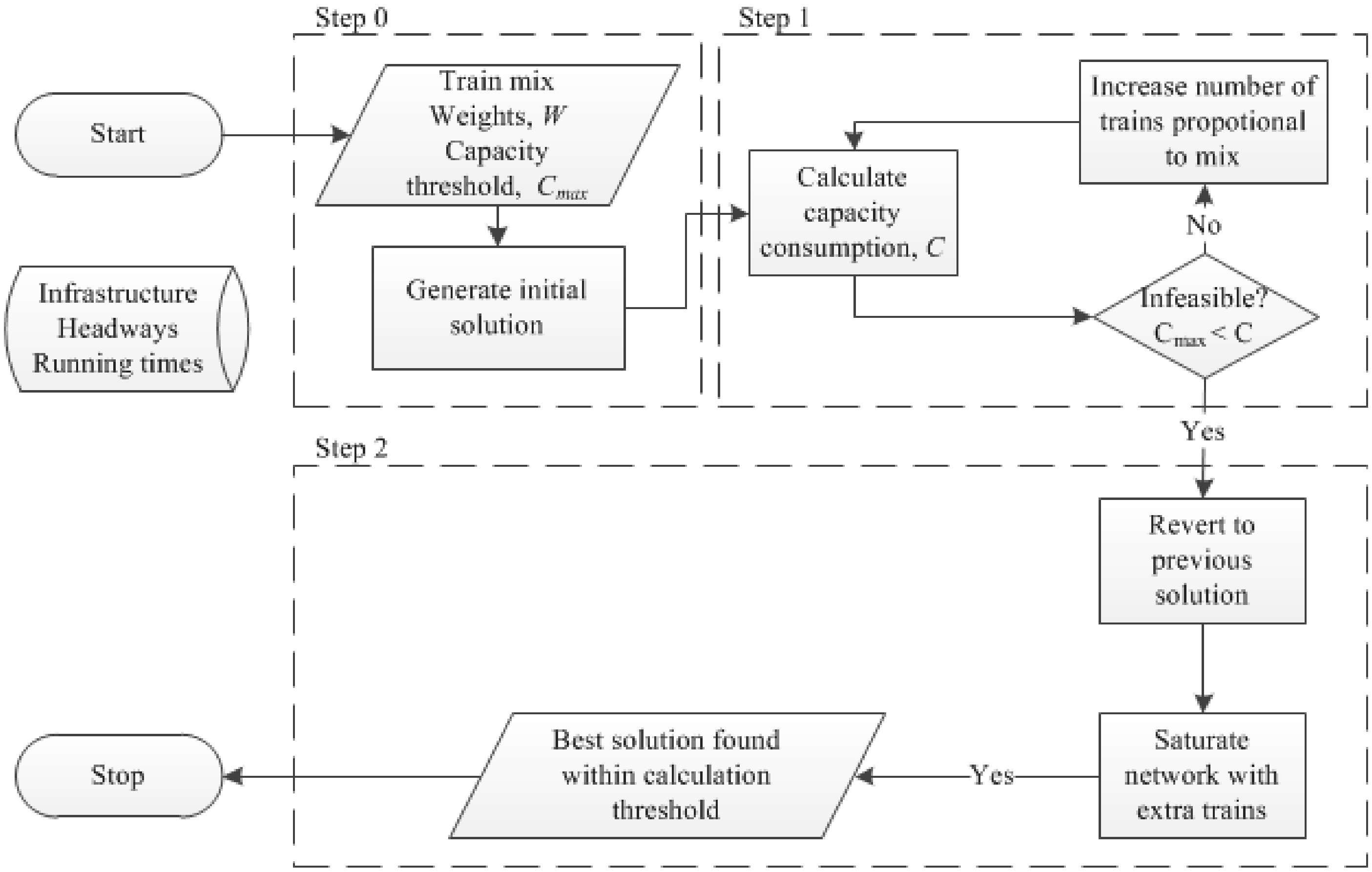
Deterministic capacity consumption

- Output
  - Cumulative distribution of capacity consumption



Framework

Optimisation framework:



Obtaining capacity span

- For the capacity consumption calculation, *C*, a percentile, *p*, is required

- Can be done for different values of *p* to obtain a span of trains:
  - 0: maximum number of trains (can only be fitted in perfect order)
  - 50: median number of trains (only for 50% of the sequences)
  - 100: minimum number of trains (can always be fitted)

Mathematical model for step 2

- Step 2: We want to add as many extra trains as possible

$$\text{maximize} \sum_{i=1}^{n} (w_i \cdot x_i)$$

s.t.

$$x_i \geq 0$$

$$C(X+Y,p) \leq C_{max}$$

Number of trains of type *i*

Weight for train type *i*

Capacity consumption of solution must not exceed threshold

$X=(x_1 \ldots x_i \ldots x_n)$

$Y=(y_1 \ldots y_i \ldots y_n)$, trains added in step 1

Percentile, *p*

p = 0: only the "best" sequence

p = 100: all sequences fit

Determining absolute railway capacity

- Simple problem, except:
  - *C(X+Y,p),* the capacity consumption, is expensive to evaluate

- Well-known within simulation-based optimisation
  - Simulation can be approximated using e.g. Response Surface Model based on regression or neural networks
  - A good approximation will provide quality solutions fast

- However, in simulation-based optimisation simulation is used for the objective function not the constraints

Solution method

- Model solved using a greedy heuristic
- A modified binary search is used to find the maximum number of trains to add of a type (*dSearch* in the code below)

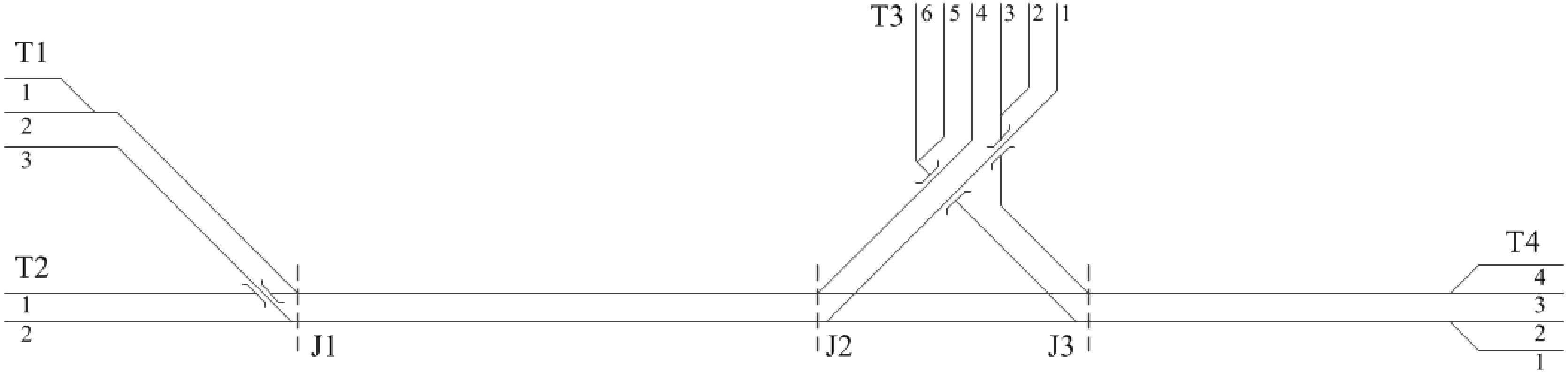
Algorithm 1: Pseudocode for greedy heuristic.

Data: Weights, $w_i \in X$
Result: $X^* = (x_1, ..., x_i, ..., x_n)$ a solution that maximize $\sum_{i=1}^{n}(w_i \cdot x_i)$
$x_i^* = 0 \quad \forall i \in \{1, ..n\}$
for $i \in X$ *in descending order of* w_i **do**
  $x_i = 0 \quad \forall i \in \{1, ..n\}$ `// Reset solution`
  $x_i = dSearch(i)$ `// Find maximum amount of trains to add`
  for $j \in X$ *in descending order by* w **do**
    if $j \neq i$ **then**
      $x_j = dSearch(j)$
    end
  end
  $X^* = max(obj\_val(X), obj\_val(X^*))$ `// Set best solution`
end
return X^*

Case

- Mesoscopic network (feasibility by microscopic input data)

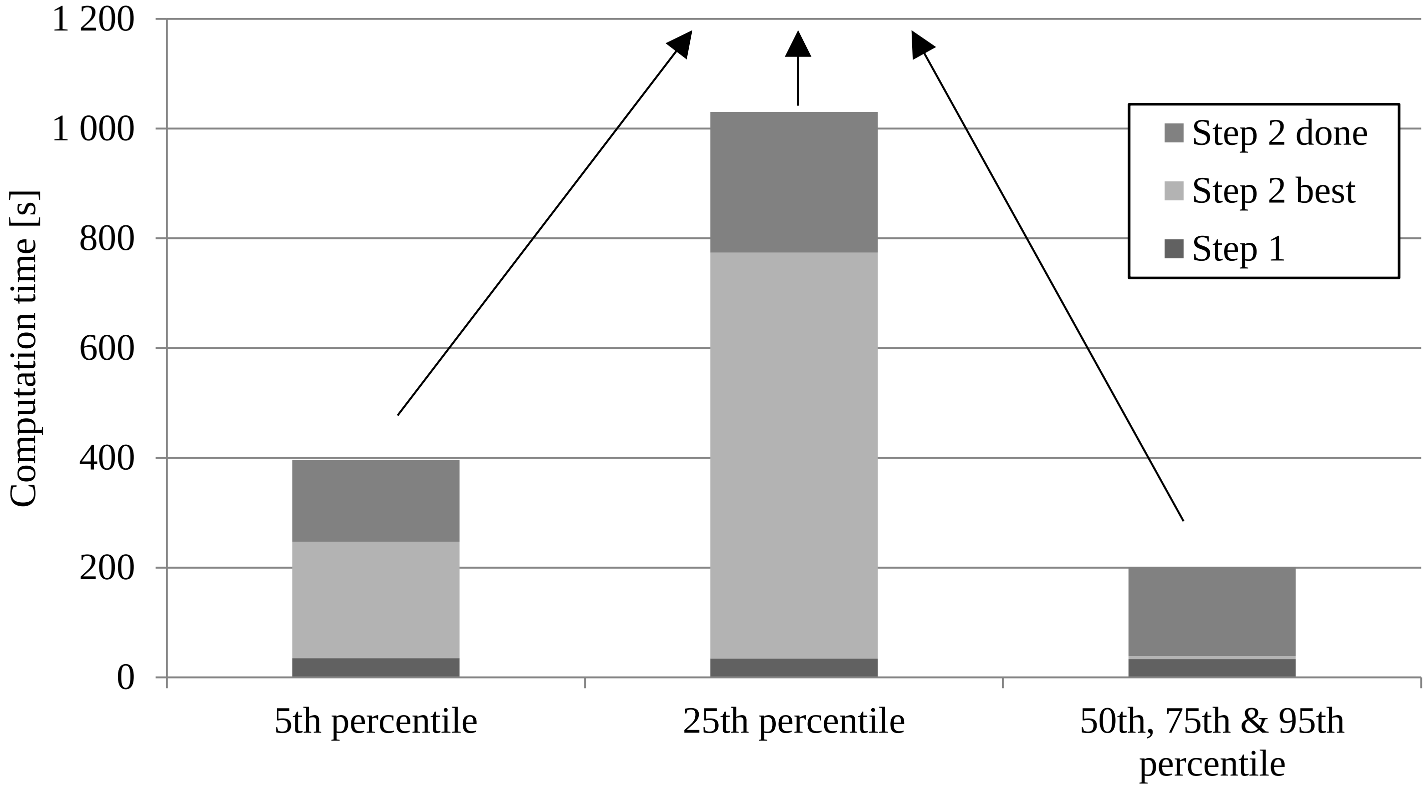
Train type	Share (step 1)	Weight (step 2)
ICE (T3↔T4)	12.5%	3
IC (T3↔T4)	12.5%	2
IC (T1↔T4)	12.5%	2
RE (T1↔T3)	12.5%	1
RE (T3↔T4)	25%	1
Freight (T2↔T4)	25%	4



Results

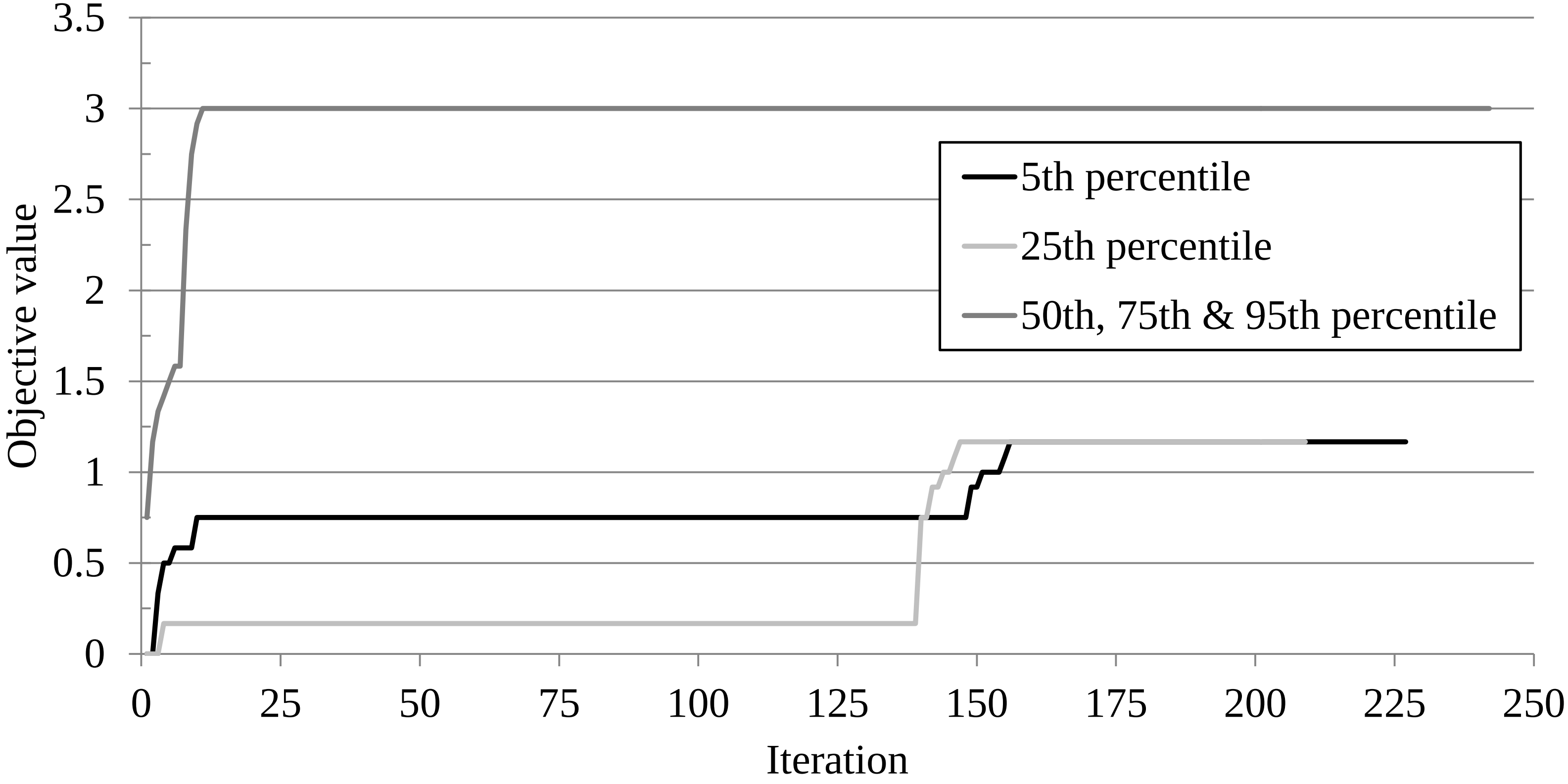
- Computational results for step 1 and 2:
  - Win 7 desktop w/ Intel Core i7-2600 (3.4 GHz) CPU and 8 GB RAM

Increased running time of *C(X+Y,p)* due to heterogeneity



Results

- Computational results for step 2



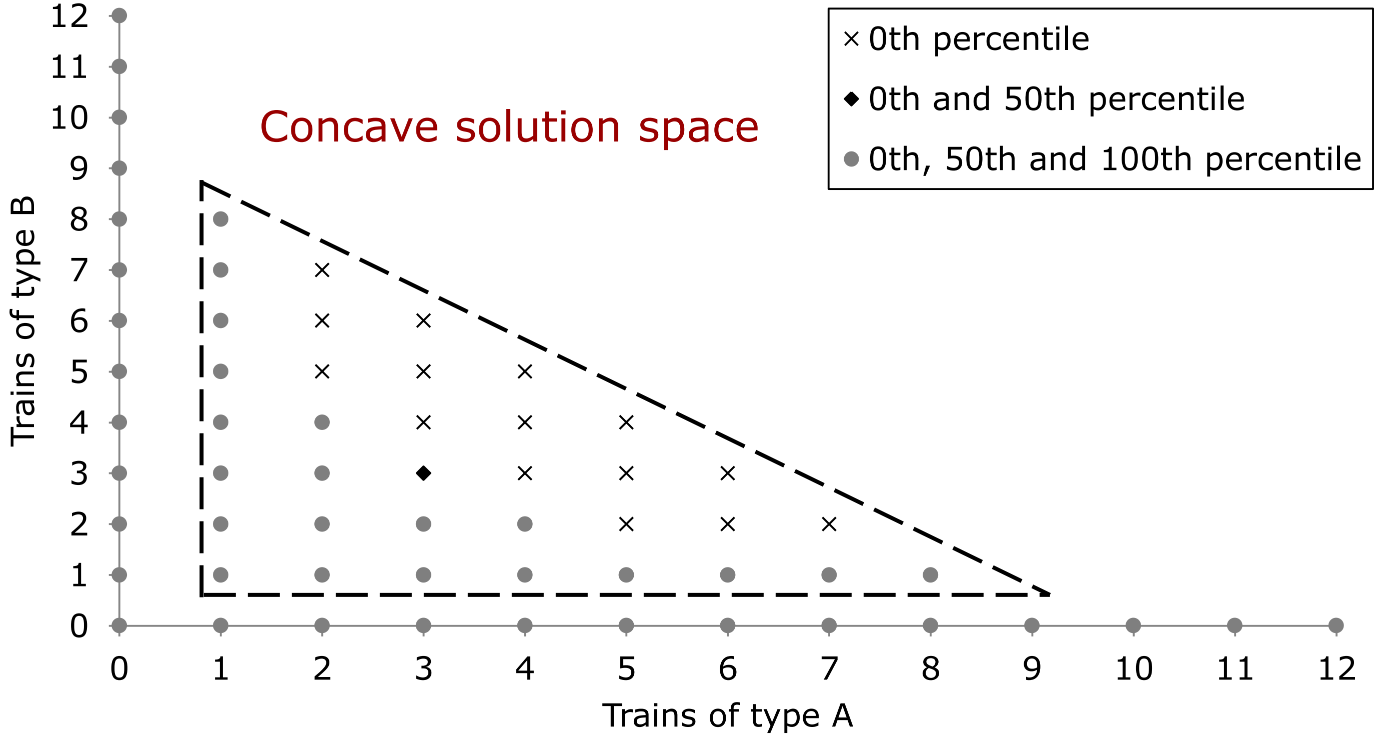
Results

- Results for case

C_{max}	75%	90%	90%	90%	100%	100%	100%
Percentiles	5-95th	50-95th	25th	5th	75-95th	50th	5-25th
Mix scheduled			•	•		•	•
Train type	Number of trains (extra trains added in step 2)						
1	0	0	1	2 (1)	0	1	2 (1)
2	0	0	1	1	0	1	1
3	0	0	1	1	0	1	1
4	0	0	1	1	0	1	1
5	0	0	1	1	0	1	1
6	0	0	1	1	0	1	1
7	0	0	1	1	0	1	1
8	0	0	1	1	0	1	1
9	0	0	16 (14)	2	0	18 (16)	2
10	0	0	2	2	0	2	2
11	16 (16)	19 (19)	2	9 (7)	22 (22)	2	11 (9)
12	18 (18)	21 (21)	2	5 (3)	24 (24)	2	7 (5)
Mix - objective	0 - 5.2	0 - 6.2	1 - 0.5	1 - 1.7	0 - 7.1	1 - 0.6	1 - 2.3

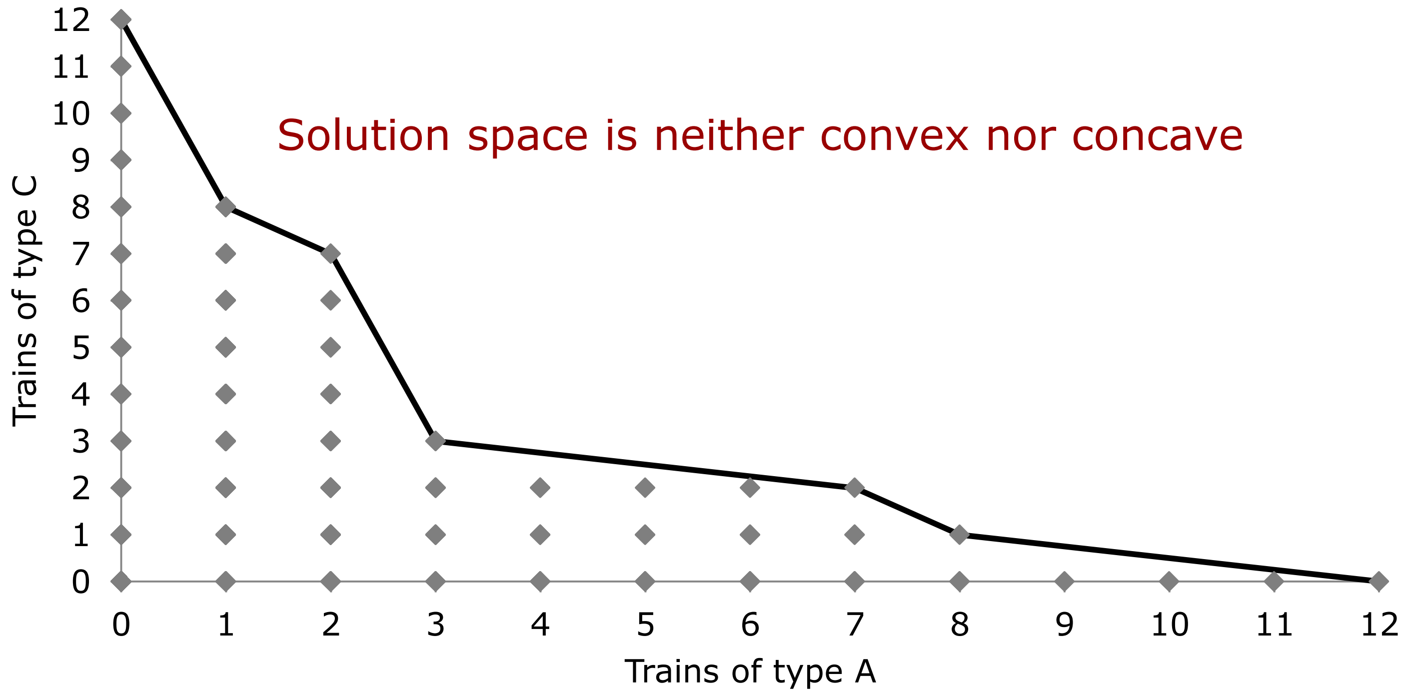
Solution space

- Does the greedy heuristic finds the optimal solution?
  - In some cases yes:



Solution space

- Does the greedy heuristic finds the optimal solution?
  - In some cases yes:
  - And in some cases no:



Conclusions

- Proposed framework determines the **number of times a certain mix of train can be scheduled plus additional trains** that saturate the mix solution using predefined train type weights

- The framework is able to give a span of capacity in networks with heterogeneous operation

- For the case considered
  - Capacity estimated in ~3 minutes (in most test instances), but in some instances up to ~20 minutes

- In step 2, as many trains as possible is added of one single type
  - Not desirable as solutions might contain a very high proportion of one train type

- Greedy heuristic does not guarantee optimal solutions

Further work

- Other solutions methods for the mathematical model (step 2)
  - Use metaheuristics to obtain better solutions

- Alternative/extended formulation of the mathematical model
  - To ensure more diversity in train types added in step 2 (not only one type)
  - To ensure directional symmetry

- Use stochastic capacity consumption model instead of deterministic
  - Will capture the aspect of robustness better and thus provide better capacity results
  - Requires some improvements in calculation time

Thank you for your attention!

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