Duty Rostering in Public Transport – Facing Preferences, Fairness, and Fatigue [1]

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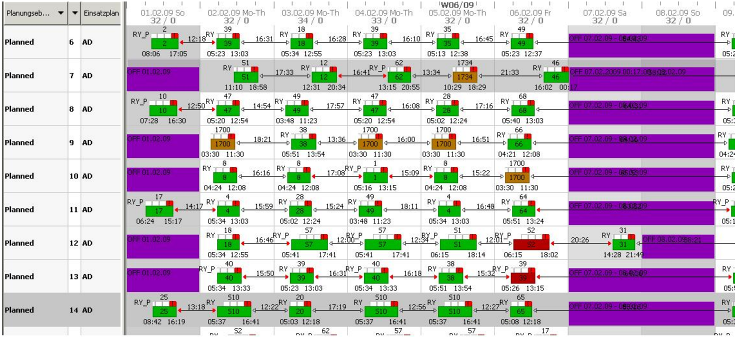
Rostering

- Assigning duties to employees obeying certain rules.

Objectives

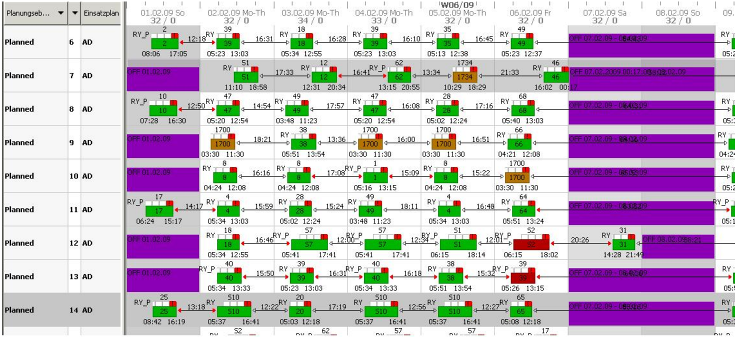
- Minimize costs
- Create employee-friendly and fair rosters



Rostering

Variants:

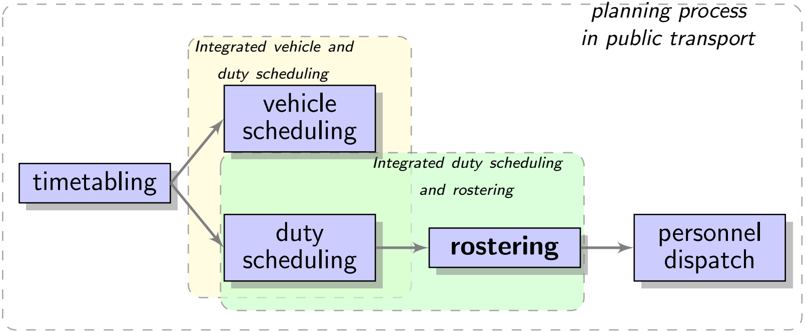
- Certain planning period or idealized days of operation
- Personalized or anonymous
- Cyclic or acyclic
- Days off planned in advance or as part of the optimization



Rostering

Planning process:

- Important step in several industries (healthcare, public transport, railway, airline, . . .)
- Sequential after duty scheduling
- Integrated with duty scheduling



Rostering Rules

Hard Rules

- Come from law or collective agreements
- Must not be violated
- Examples: Weekly Rest Time

Weekly Rest Time

In any 2 consecutive weeks the driver must

Take at least:

- 2 regular rest periods (of at least 45 hours)

Or

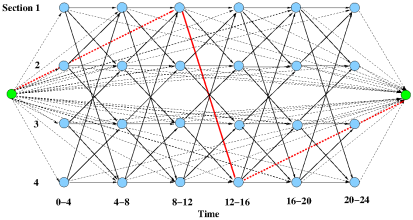
- 1 regular rest period and 1 reduced rest period (of at least 24 hours)

Soft Rules

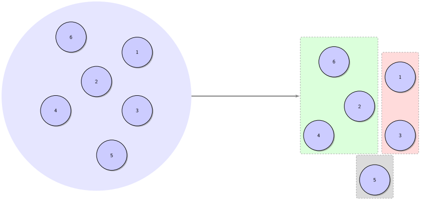
- Violation is penalized by costs
- Can be used to compute employee-friendly rosters
- Example: Maximize number of complete free weekends

Rostering Models

- Multi-Commodity Flow



- Set Partitioning



Multi-Commodity Flow Model

Multi-Commodity Flow Model

- A duty is a node in a graph
- Arc (i, j) means, duties i and j can be performed subsequently by the same employee
- One commodity per employee
- Additional resource constraints

Flow Model more effective

- Many feasible paths
- Only a few hard rules

Multi-Commodity Flow Model

- D: Set of duties
- M: Set of rows.
- $\mathcal{I}$: Set of set of arcs
- I: Subset of all arcs.
- Graph G: One node per duty plus s and t as source and sink.
- A: Set of arcs of G.
- R: Set of resources.

$$
\begin{aligned}
\min \sum_{m\in M}\sum_{a\in A} c_{am}x_{am} & & \text{(FROSTER)} & \\
\sum_{m\in M}\sum_{a\in\delta^{\text{in}}(v)} x_{am} &= 1, & \forall v\in D, & \quad \text{Cover the duties}\\
\sum_{a\in\delta^{\text{in}}(v)} x_{am} - \sum_{a\in\delta^{\text{out}}(v)} x_{am} &= 0, & \forall m\in M, \forall v\in D, & \quad \text{Every row is a path in the network}\\
\sum_{a\in A} b_{ar}x_{am} &\le u_{rm}, & \forall m\in M, \forall r\in R, & \quad \text{Resource constraints}\\
\sum_{a\in I} x_{am} &\le |I|-1, & \forall I\in\mathcal{I}, \forall m\in M, & \quad \text{Forbid infeasible sequences of arcs}\\
x_{am} &\in \{0,1\}, & \forall a\in A, \forall m\in M. &
\end{aligned}
$$

Set Partitioning Model

Set Partitioning Model

- One variable per roster
- Rosters must be generated
- Only feasible rosters are generated
- Infeasible rosters are not part of the model

Set Partitioning Model more effective when

- Many hard rules

Set Partitioning Model

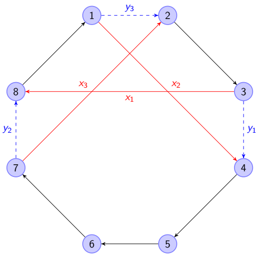
- D: Set of duties.
- P: Set of rows.

$$
\begin{aligned}
\min \sum_{p \in P} d_p y_p & & \text{(CRoster)} & \\
\sum_{p \ni d} y_p = 1, & & \forall d \in D, & \quad \text{every duty is covered by one row} \\
\sum_{p \in P} e_{rp} y_p \leq u_r, & & \forall r \in R, & \quad \text{Resource constraints} \\
y_p \in \{0,1\}, & & \forall p \in P. &
\end{aligned}
$$

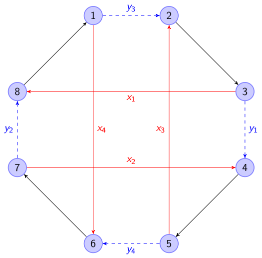
Problem Cannot solve big instances directly in an acceptable time.

DEX (Dynamic-depth-EXchange heuristic)

- Multi-phase-heuristic for rostering
- Steps:
  1. Construct a roster scheme
  2. Chain k-opt moves with $k \in 1, ..., 4$ to large alternating cycles, i.e., Lin-Kernighan search



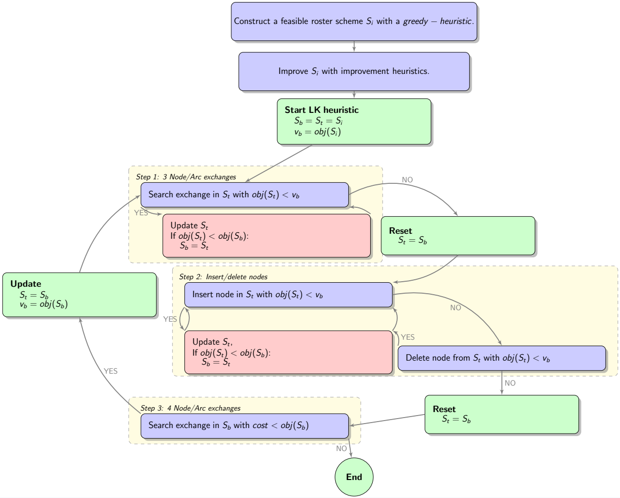
Three Edge Exchange



Four Edge Exchange

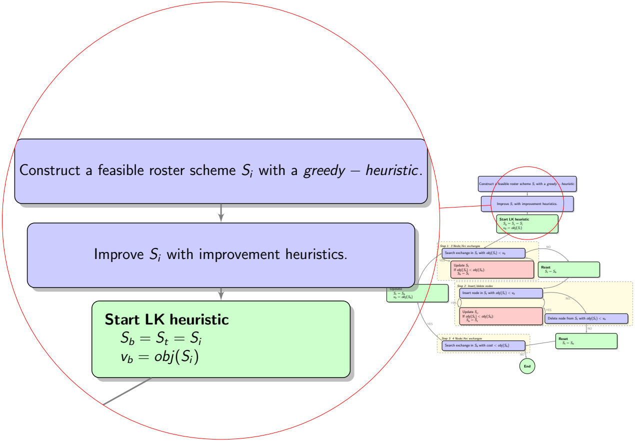
DEX-Heuristic

- S_b: Best solution.
- S_t: Temporary solution.



DEX-Heuristic Start

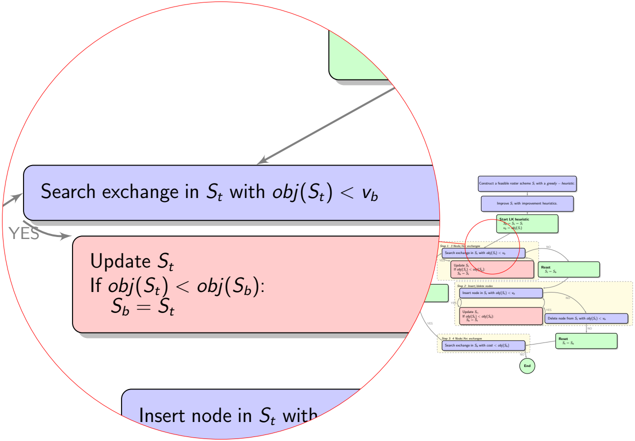
- Start solution with greedy heuristic
- Improve solution with k-opt steps



DEX-Heuristic Improvement

LBW
Mathematical Optimization Systems in Public Transit

- Variable depth search
- i-Three edge exchanges correspond to a $2 * (i - 1) + 3$ edge exchange



Rostering Applications at Zuse-Institute Berlin

- Rostering in Toll Enforcement.
- Cyclic rostering in public transport.
- Airline Crew Rostering.

Rostering Application - Toll Enforcement

Toll for Trucks on German Motorways

- Distance-based toll for trucks on German motorways.
- Rates per kilometre differ between 0.12 - 0.22 EUR/km.
- On-Board-Units (OBU) recognize a toll road by GPS on their own.
- Trucks without OBUs require manual toll booking.

Toll Enforcement Problem (TEP)

Problem Input

- Personalized data
- Resources

Problem Output

- Schedules for all mobile control teams

Two parts

- Tour Planning / duty scheduling
- Crew assignment / rostering

Optimisation Goals

- Network-wide control
- Consider spatial and temporal distribution of truck traffic

TEP Model

- Extend (FRoster) with tour planning
- J: Set of days
- F: Set of control groups.
- $z_d, d \in D$: Decide if a control tour d is chosen or not.

(Objective)

$$\max \sum_{d \in D} w_d z_d - \sum_{m \in M} \sum_{a \in A} c_{am} x_{am}$$

(Coupling Constraints)

$$n_d z_d - \sum_{m \in M} \sum_{a \in \delta^{\text{in}}(d) \in A} x_{am} = 0, \ \forall d \in D$$

(Tour planning)

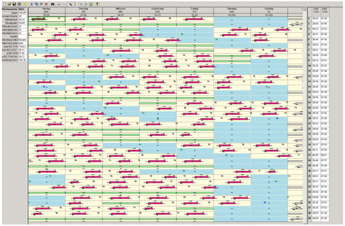
$$\begin{aligned}
\sum_{d \in D_f \cap D_j} z_d &\leq 1, && \forall f \in F, \forall j \in J, \text{(one tour per day)} \\
\sum_{d \in D_s \cap D_j \cap D_i} z_d &\leq 1, && \forall s \in S, \forall j \in J, \forall i \in T \text{(one group per section)} \\
\sum_{d \in D_s} \beta_{ds} z_d &\geq \kappa_s, && \forall s \in S, \text{(minimum control frequency)} \\
z_d &\in \{0,1\}, && \forall d \in D.
\end{aligned}$$

(FRoster)

$$\begin{aligned}
\sum_{a \in \delta^{\text{in}}(v)} x_{am} - \sum_{a \in \delta^{\text{out}}(v)} x_{am} &= 0, && \forall m \in M, \forall d \in D, \\
\sum_{a \in A} b_{ar} x_{am} &\leq u_{rm}, && \forall m \in M, \forall r \in R \\
\sum_{a \in I} x_{am} &\leq |I| - 1, && \forall I \in \mathcal{I}, \forall m \in M \\
x_{am} &\in \{0,1\}, && \forall a \in A, \forall m \in M,
\end{aligned}$$

Cyclic Rostering

- To receive equitable rosters, the duties are scheduled cyclically
- Every employee conducts the same duties
- Transparent for Trade Unions



Cyclic Rostering Problem (RPT)

Problem Input

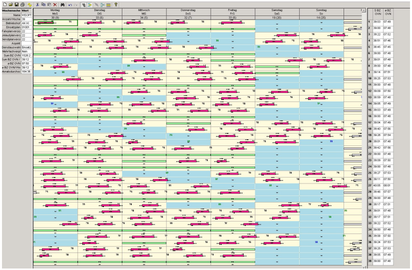
- Duties
- Rostering rules

Optimisation Goals

- Minimize number of rows
- Reach a uniform distribution of paid time
- Fair distribution of unpopular duties (e.g. split duties, night duties)
- Other criteria of fairness

Problem Output

- Cyclic roster



RPT Model

- Extend (CRoster) with ATSP Constraints
- $y_{rs} r, s \in P$: Variables for successors and predecessors.

(Objective)

$$\min \quad \sum_{p \in P} c_p x_p + \sum_{r,s \in P} d_{rs} y_{rs}$$

(ATSP)

$$\sum_{s \in P} y_{rs} = x_r \qquad \forall r \in P (\text{one successor})$$
$$\sum_{s \in P} y_{sr} = x_r \qquad \forall r \in P (\text{one predecessor})$$
$$\sum_{r \notin S, s \in S} y_{rs} \geq x_p + x_q - 1 \forall p \notin S, q \in S, S \subset P (\text{subtour elimination})$$
$$y_{rs} \in \{0,1\} \qquad \forall r, s \in P$$

(CRoster)

$$\sum_{p \ni d} x_p = 1 \qquad \forall d \in D$$
$$\sum_{r \in \sigma_i} b_{ri} x_r \in [l_i, u_i] \quad \forall \sigma_i \in S_i, i \in I$$
$$x_p \in \{0,1\} \qquad \forall p \in P$$

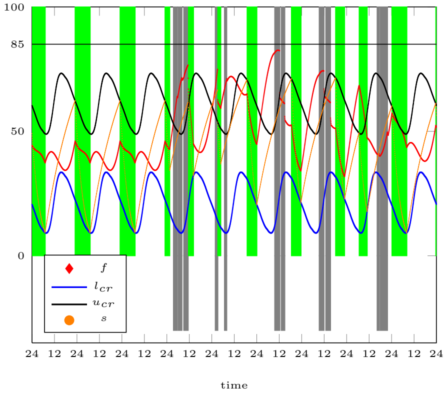
Rostering Application - Airline Crew Rostering

Biorhythm of crew members

- During sleep one recovers
- Staying awake makes one tired
- Working makes one tired even faster
- *No one can sleep on command*

Maximum fatigue rules

- Complex non linear function
- Taking time zones into account



RPT Szenario

Medium urban scenario

- 157 duties
- Target working time 39 hours per week
- Regular weekly rest time of 45 hours
- Short weekly rest time of 36 hours

Optimisation Goals

- Working time per week as close to 39 hours as possible
- Minimize the number of short weekly rests
- Maximize the number of free weekends (Saturday and Sunday free)
- Minimize the number of stand alone duties (Free - Duty - Free)

RPT Results with DEX

Aspect	Manual solution	DEX
Employees	39	39
PT/Week (optimal 39:00)	[37:56,40:11]	[38:45,40:05]
Separated weekend duties	12	9
Free weekends	12	16
Stand-alone duties	0	0
Number of short frees	13	6
Runtime (hh:mm)	-	00:22

Results

- Working time per week is closer to 39 hours
- Number of short frees decreases from 13 to 6
- Free weekends increases from 12 to 16

TEP Szenarios

- Nine scenarios from seven regions
- Regions are optimized separately at BAG

Instance	Region	Inspectors	Sections	Fixed Duties	Duty Types	IP Rows	IP Columns
I1	r_1	21	17	253	6	7738	96526
I2	r_1	22	22	272	4	8010	101791
I3	r_1	22	22	170	7	13095	392563
I4	r_2	23	24	189	8	15417	402285
I5	r_3	22	22	8	12	20366	1611980
I6	r_4	19	17	177	8	11246	295388
I7	r_5	23	19	182	9	15067	501340
I8	r_6	24	28	57	8	15246	712228
I9	r_7	21	16	0	10	17369	904878

- TEP is directly solvable by a solver such as CPLEX

TEP Test settings

Algorithm

- Enumerate all possible tours
- Start the DEX heuristic

CPLEX settings

- Time limit of five minutes to compare results with DEX
- Time limit of 12 hours to check if CPLEX finds better or optimal solutions

Results

- For instances I1, I2 and I6 the IP method outperforms DEX
- All others have no feasible solution within five minutes
- DEX always finds a feasible solution
- DEX is the first choice for medium-size and large instances

	IP			DEX		
instance	obj 5 min	obj 12 hours	gap(%)	obj	time(s)	gap(%)
I1	359,077.13	359,077.13	0.00	353,181.93	36	1.67
I2	332,283.74	332,556.68	0.00	326,612.72	82	1.82
I3	–	513,998.85	0.00	499,804.91	127	2.84
I4	–	346,788.20	0.88	340,130.17	179	2.85
I5	–	805,294.03	1.08	790,459.06	420	2.98
I6	154,142.04	154,270.86	0.03	151,769.96	151	1.68
I7	–	335,015.01	0.72	331,301.40	162	1.85
I8	–	373,509.57	85.84	671,729.68	415	3.33
I9	–	437,426.84	4.76	441,274.11	474	3.85

Thank you for your attention!

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