Tariff Zone Planning for Public Transport Companies

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Schedule

Tariff Systems

- **Distance tariff**: price depends on trip length
- **Unit tariff**: all trips cost the same
- **Zone tariff**:
  - *arbitrary prices*: arbitrarily chosen price for a given pair of zones (orig-dest)
  - *counting zones*: number of zones touched on trip times price per zone

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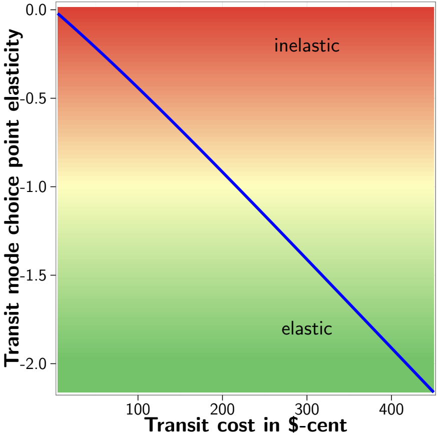
Revenue Management for Public Transport Companies

- Revenue maximization
- Optimization of prices and zone structure
- Demand is a function of price and travel time

Public Transport Demand (Example SF Bay Area)

- Transit mode choice is price (in-) elastic (f.e., Hensher (2010), *TRB*)



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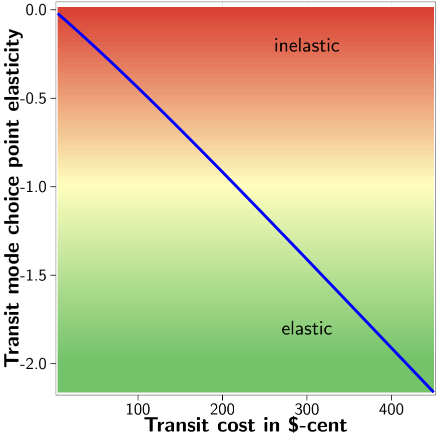
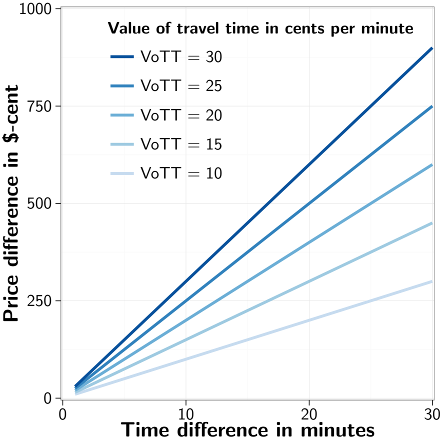
- Transit mode choice is price (in-) elastic (f.e., Hensher (2010), *TRB*)
- Customers choose time shortest path (f.e., Noland/Polak (2002), *TranRev*)





Object of Research

Problem Statement

Partition the service area into *zones* and find a *fare* (price per touched zone) such that the total expected revenue is maximized.

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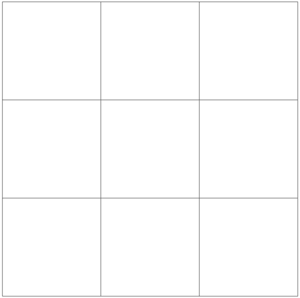
Issues

- Focus on conversion to tariff zone system (given reference price)
- Demand is exogenous and static: demand independent from price
- Customers choose cheapest path

Concept

Public Transport Graph

- Service area divided into districts



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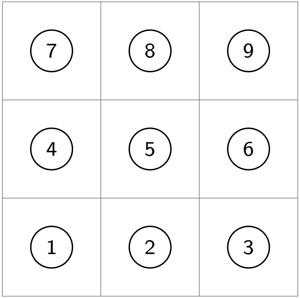
- One unique (artificial) stop per district: set $\mathcal{I}$



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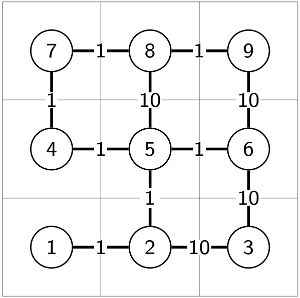

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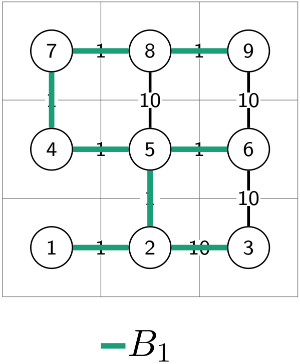
B_o Shortest path tree with root $o \in \mathcal{I}$



Zones and Zone Borders

Set

t Number of zones on shortest path from i to j with $t = 1, \ldots, T_{ij}$

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Variables

W_{ij} Number of zone borders $(t-1)$ on shortest path from $i \in \mathcal{I}$ to $j \in \mathcal{I}$

X_{ij} =1, if a zone border exists between adjacent stops i and j, i.e. $[i,j] \in \mathcal{E}$

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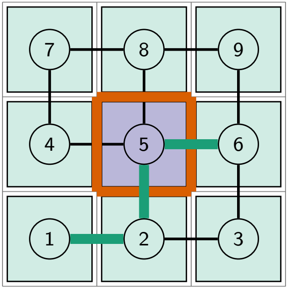

Example:

$X_{2,5} = X_{5,6} = 1$
$\Rightarrow W_{1,6} = 2$ zone borders
$\Leftrightarrow t = 3$ zones on shortest path 1-6

Revenue and Zones

Fare

- π Price per zone touched on shortest path
- Actual ticket cost (single, monthly etc.) derived from π

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Transit Demand

$\text{PTT}_{ijt}(\pi)$ Number of public transport trips given fare π and $t = 1, \ldots, T_{ij}$ touched zones on shortest path i-j

$$\text{PTT}_{ijt}(\pi) = f(\pi \cdot t, \text{travel-time}, \ldots)$$

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Revenue

$r_{ijt}(\pi)$ expected revenue on shortest path i-j given fare π and $t = 1, \ldots, T_{ij}$ touched zones

$$r_{ijt}(\pi) = \pi \cdot t \cdot \mathsf{PTT}_{ijt}(\pi)$$

Revenue and Zone Borders

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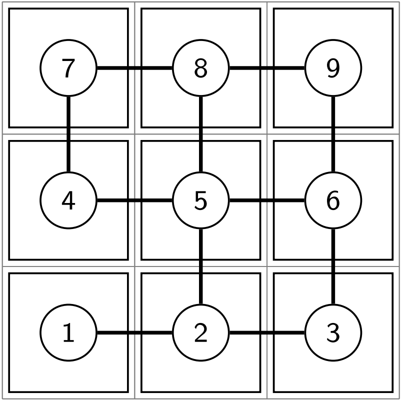
Y_{ijt} =1, if $t = 1, \ldots, T_{ij}$ tariff zones are touched on shortest path i-j



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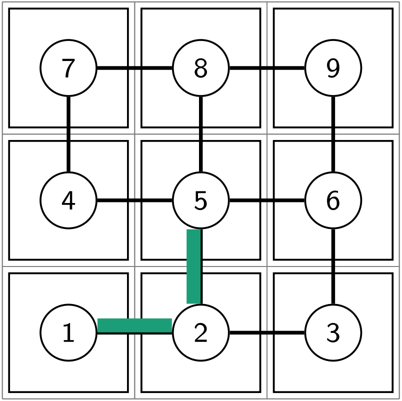

- $i = 1$ to $j = 5$

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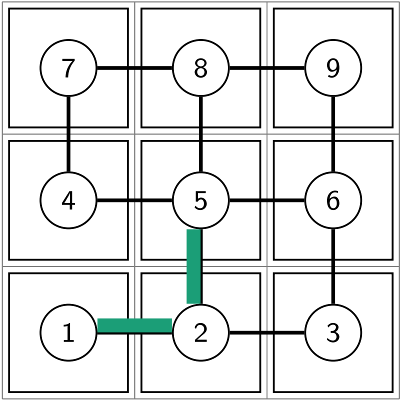

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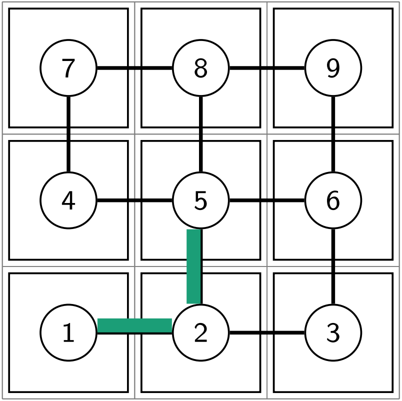

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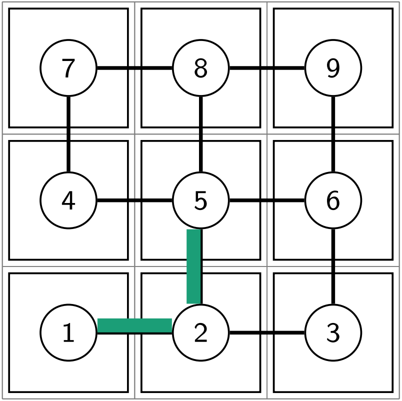

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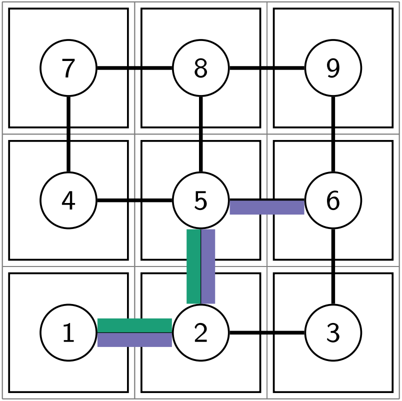

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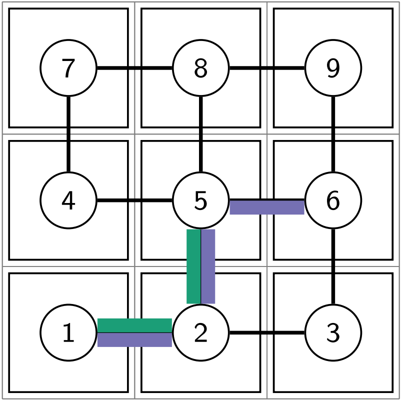

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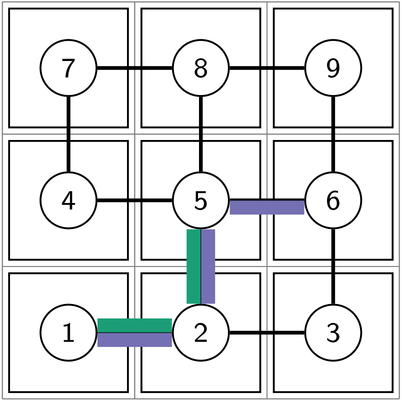

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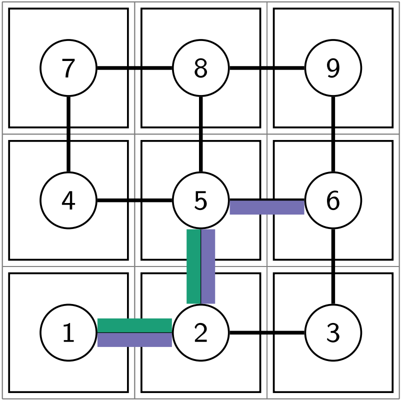

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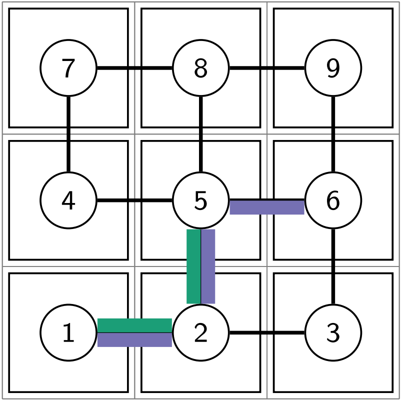

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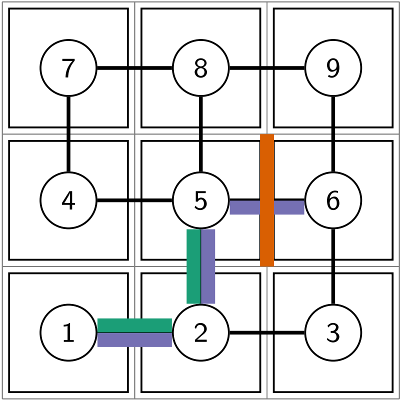

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Tariff Zone Contiguity

Contiguity

All stops of a tariff zone are surrounded by a continuous tariff zone border. I.e., all stops within a tariff zone can be reached without crossing a tariff zone border

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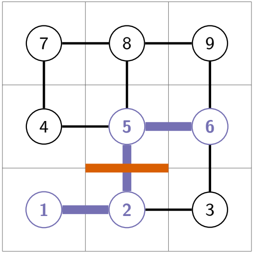

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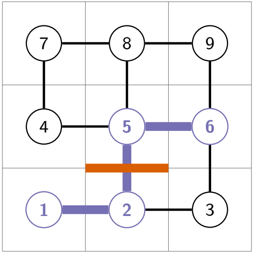

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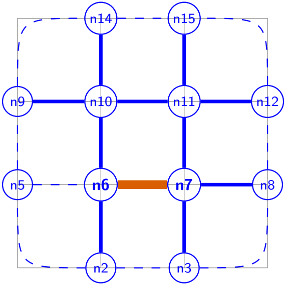

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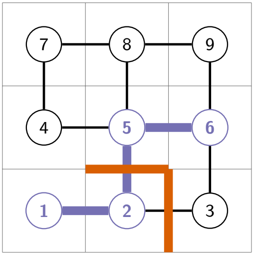

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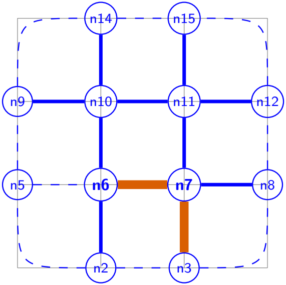

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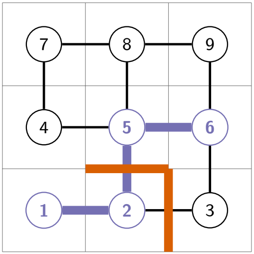

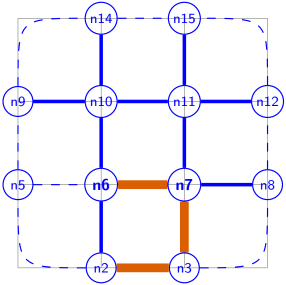

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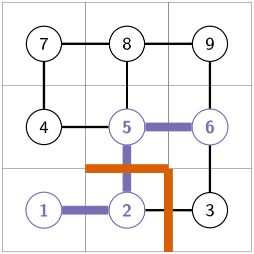

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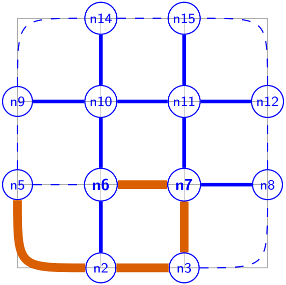

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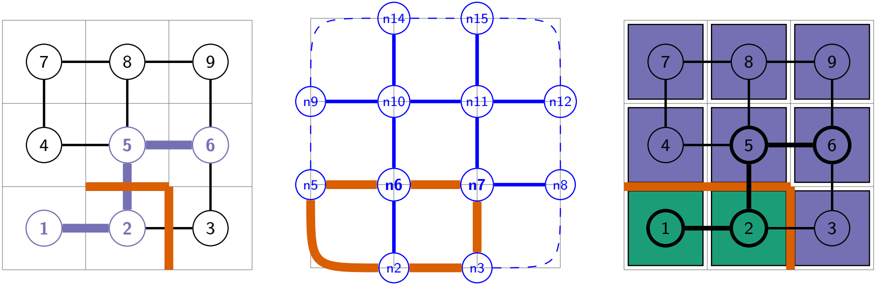

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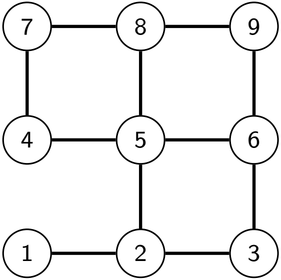
Enforcing Structure on District Borders

Input data



Feasible solutions

Enforcing Structure on District Borders

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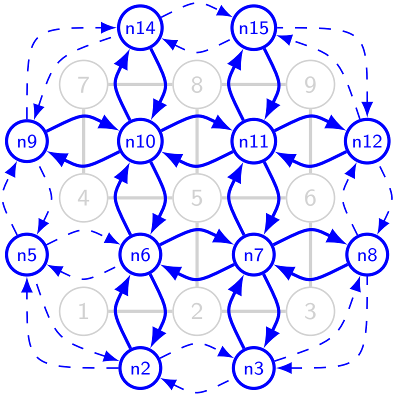

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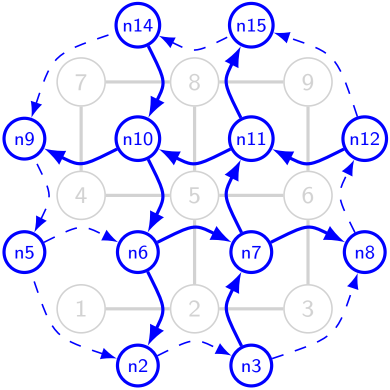

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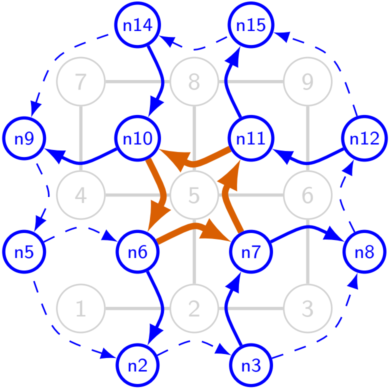

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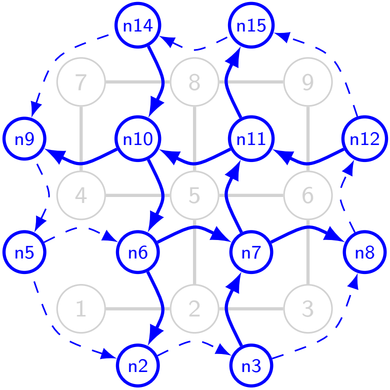

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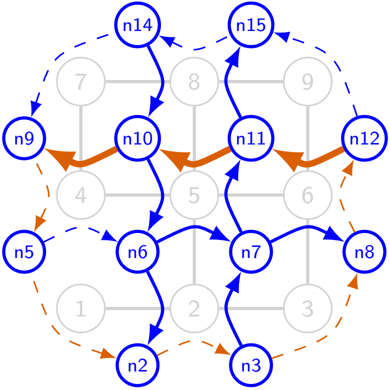

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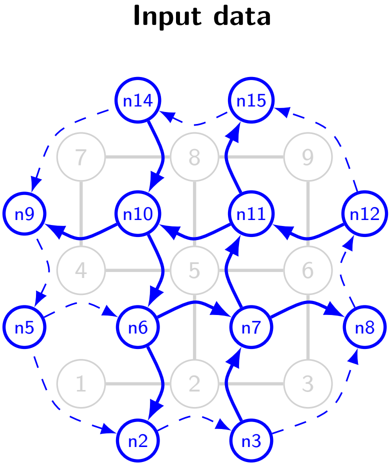

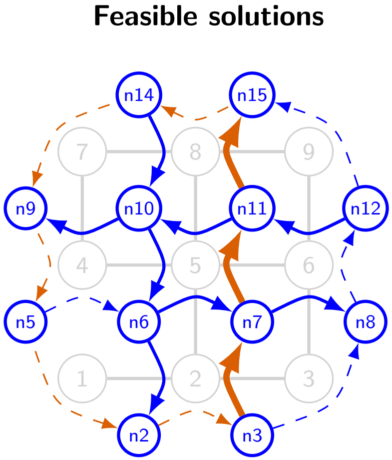

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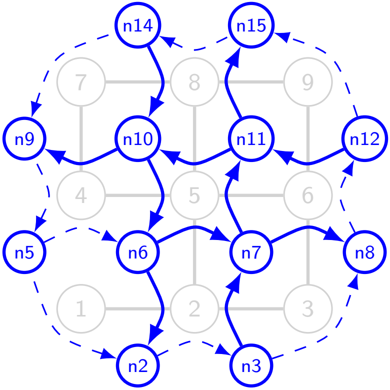

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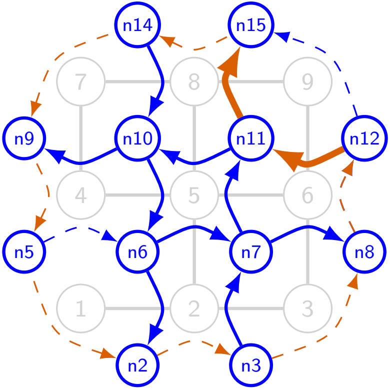

Tariff Zone Planning Problem

Model Part I: Maximizing Revenue

Objective: revenue maximization for a given π

$$\text{Maximize } F(\pi) = \sum_{\substack{i,j \in \mathcal{I},\\ i<j}} \sum_{t=1}^{T_{ij}} r_{ijt}(\pi)\, Y_{ijt} \tag{1}$$

Note, $Y_{ijt} \in \{0, 1\}$

Definite number of tariff zones on shortest path i-j

$$\sum_{t=1}^{T_{ij}} Y_{ijt} = 1 \qquad \forall\, i, j \in \mathcal{I}, i < j \tag{2}$$

Model Part II: Zones and Zone Borders

Determine number of zone borders on shortest path i-j

$$\sum_{t=1}^{T_{ij}} (t-1)\, Y_{ijt} = W_{ij} \qquad \forall\; i, j \in \mathcal{I}, i < j \tag{3}$$

Note, $W_{ij} \geq 0$

Model Part II: Zones and Zone Borders

Determine number of zone borders on shortest path i-j

$$\sum_{t=1}^{T_{ij}} (t-1)\, Y_{ijt} = W_{ij} \qquad \forall\, i,j \in \mathcal{I}, i < j \qquad (3)$$

Note, $W_{ij} \geq 0$

Location of zone borders in shortest path tree

$$W_{oi} + X_{ij} + X_{ji} = W_{oj} \qquad \forall\, o \in \mathcal{I}, [i,j] \in B_o \qquad (4)$$

Note, $W_{ii} = 0$ and $X_{ij} \in \{0, 1\}$

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Symmetry

$$X_{ij} + X_{ji} \leq 1 \qquad \forall\ [i,j] \in \mathcal{E}, i < j \qquad (5)$$

Model Part III: Contiguity

Coupling zone border (between i and j) and flow (along n, m)

$$X_{ij} \leq V_{nm} \leq \alpha X_{ij} \qquad \forall \ [i,j] \in \mathcal{E}, [n,m] \in A_{[i,j]}, i < j \tag{6}$$

$$X_{ji} \leq V_{mn} \leq \alpha X_{ji} \qquad \forall \ [i,j] \in \mathcal{E}, [n,m] \in A_{[i,j]}, i < j \tag{7}$$

Notes

- $A_{[ij]}$ Arc of district border corresponding to edge $[i,j]$ of the transit graph
- V_{nm} Non-negative flow variable corresponding to arc $[n,m]$ of the district border graph G
- α Sufficiently large scalar

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Flow constraints

$$\sum_{[n,m] \in G} V_{nm} - \sum_{[m,n] \in G} V_{mn} = 0 \qquad \forall \; n \in N \tag{8}$$

N Nodes of the district border graph G

Notes

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Model Simplification: District Borders Structure

Replacement of (5)–(7), i.e., symmetry and flow-border coupling by

$$V_{nm} = X_{ij} \qquad \forall \ [i,j] \in \mathcal{E}, [n,m] \in A_{[i,j]}, i < j \tag{9}$$

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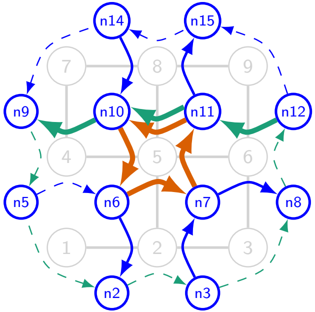
Constraining flow along a district border arc

$$\sum_{[n,m] \in G} V_{nm} \leq \alpha \qquad \forall \ m \in N \qquad (10)$$

For example, $\alpha = 1$ avoids a flow quantity of 2 along border arc $n10 - n11$



Fare Problem

Problem Statement

Find fare π, such that total expected revenue $F(\pi)$ is maximized.

- π: consider easily communicable prices. F.e. $\pi = 1, 1.1, 1.2, ..., 5$.
- Note: price degression can be easily considered

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Determine π

1. Set π to lowest/highest value
2. Solve tariff zone planning problem (1) - (10)
3. Fix lower bound on F as $\underline{F} = F^*(\pi)$
4. Set π to the next higher/lower value
5. Repeat 2 - 5 for all feasible values of π
6. Choose highest $F^*(\pi)$.

Numerical Studies and Applications

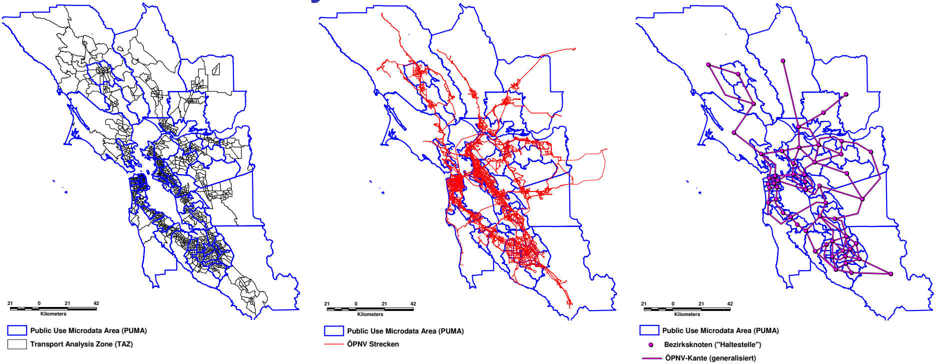
Numerical Studies: Example "Ring Structure"

- Synthetic data
- GAMS/CPLEX; standard Windows-Notebook
- CPU $\leq$ 2h per problem
- 10 different values for π
- Averages over ten random instances

$\lvert\mathcal{I}\rvert$	connectivity	total CPU	avg. gap %	% opt
81	low	256.42	0	100
81	high	448.43	0	100
121	low	1,787.72	0	100
121	high	1,075.86	0	100
169	low	7,839.24	0.64	10
169	high	7,477.88	0.45	20
225	low	9,413.61	0.02	90
225	high	15,065.79	1.24	0

Application: San Francisco Bay Area

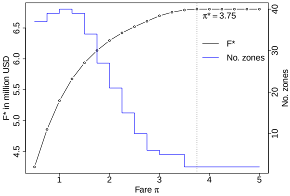
San Francisco Bay Area



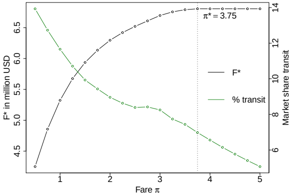
- Trips, travel-times, and travel-cost given at TAZ-level (Metropolitan Transport Commission (2008): Transportation 2035 Plan for the San Francisco Bay Area)
- Nearly 1500 TAZ $\rightarrow$ aggregation to PUMA
- Public transport graph on PUMA level: 60 nodes and 100 edges
- 20 values of π
- Transport demand model given by MTC
- **Scenario**: Unified tariff system for the entire bay area

Results

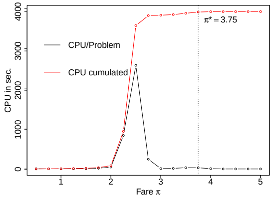
revenue & # zones



revenue & mkt. share



comp. effort



Basic scenario 2015 (MTC):

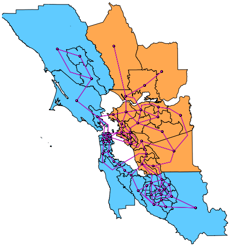
- Market share transit: 10%
- Revenue: 5.6 millionen USD/day
- Average price: 2.24 USD

Revenue maximization:

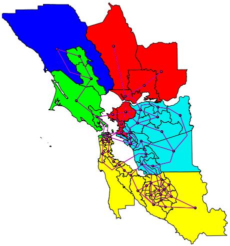
- Market share transit: 7%
- Revenue: 6.8 Millionen USD/day
- Average price: 3.89 USD

Results: Tariff Zone Maps

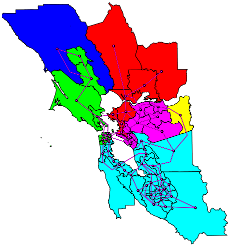
Demand maximization with respect to deviation from $F^*(\pi^*)$: δ



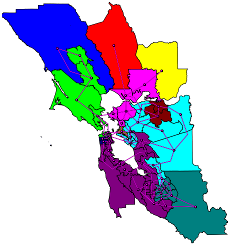
δ=0.0, transit 6.8%



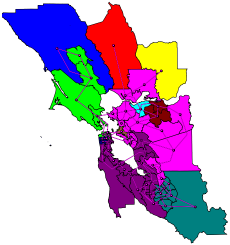
δ=0.1, transit 9.3%



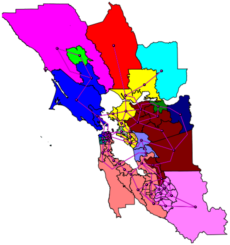
δ=0.2, transit 10.7%



δ=0.3, transit 11.4%



δ=0.4, transit 12.4%



δ=0.5, transit 13.2%

Summary & Outlook

Conclusion

- New problem
- Flexible structure
- Real world instances solvable by standard solver

Conclusion

- New problem
- Flexible structure
- Real world instances solvable by standard solver

What's next?

- Experiments on distict border structure
- Fix-and-Optimize heuristic
- Consideration of revenue sharing between service operators
- Application to road usage charging

Contact

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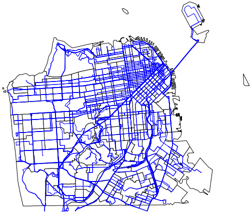
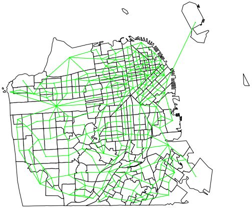
San Francisco MUNI Data

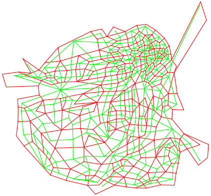
- 200 nodes, 380 edges, 20 values of π



Transit routes



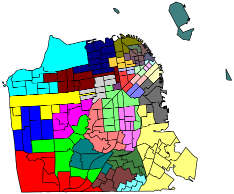
Gen. transit network



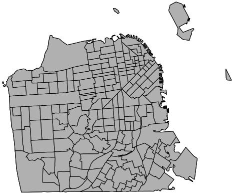
Dual graph

San Francisco MUNI Results

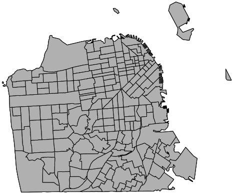
- Computational effort varies between seconds and several hours per problem



π 1 USD, 33 zones



π 2 USD (current situation)



π^* 2.8 USD