Integrated railway schedule planning

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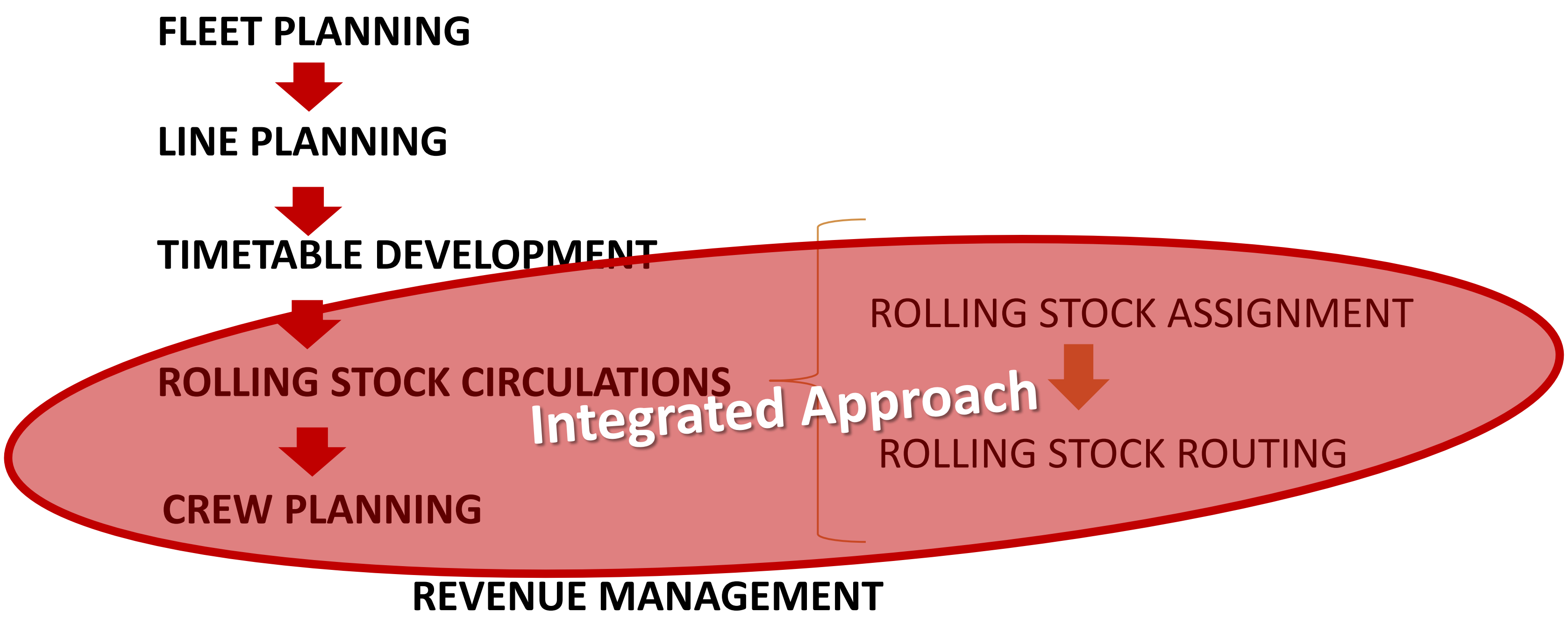
Outline

1. Introduction
2. Rolling Stock Assignment
3. Train Routing
4. Crew Scheduling
5. Integrated Approach
6. Solution Approach
7. Case Study

Introduction

Rapid Transit Railways profitability is critically influenced by its ability to construct profitable schedules.



Introduction

Rolling Stock Assignment: among the most important factors that determine cost.

Train Routing: delay minimising assignment of fleets to operations.

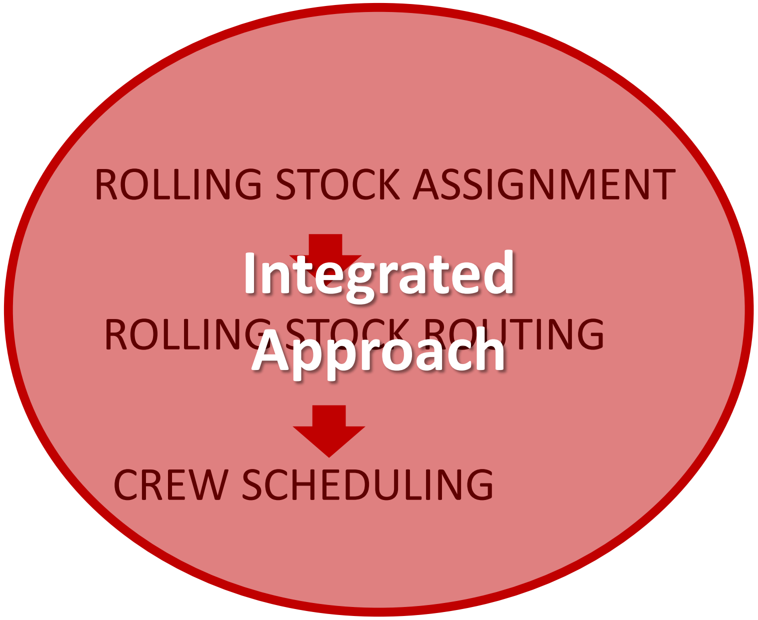
Crew Scheduling: cost minimising assignment of crews to operations.

Current practice: sequential solving.

Integrated models: better feedback and choice of schedule and capacity.

Joint optimization big challenge:

- difficult to obtain consistent data.
- Results in huge problem sizes.



Contributions

- Development of an **integrated schedule optimisation** model that includes rolling stock assignment, train routing and crew scheduling decisions.

- Introduction of **robustness** in the integrated model through different approaches (i.e., penalisation of difficult shunting operations, minimisation of propagated delays and minimisation of the need for human resources);

- Use of a **heuristic** based on Benders Decomposition so as to obtain good quality solutions;

- Development of case studies using realistic problem instances obtained from the network of the Spanish train operator RENFE.

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Rolling Stock Assignment

Optimisation of train services' compositions, empty trains and the optimal management of train units in depot stations, all while satisfying passenger demand.

Robustness: minimise *difficult* shunting operations.

Rolling Stock Assignment: Model Formulation

$$\min z = \sum_{\ell \in L} \sum_{c \in C} oc_c km_\ell x_\ell^c + \sum_{\ell \in L^t} \sum_{a \in A_\ell} upc_{a,\ell} up_{a,\ell} + \sum_{s \in SC} \sum_{c,c' \in C} \vartheta_s \cdot cc_s^{c,c'}$$

Objective function

$$\sum_{c \in C} q_c x_\ell^c \geq pf_{a,\ell} - up_{a,\ell} \qquad \forall \ell \in L^t, a \in A_\ell$$

Demand

$$\sum_{c \in C} x_\ell^c = 1 \qquad \forall \ell \in L^t$$

$$\sum_{c \in C} x_\ell^c \leq 1 \qquad \forall \ell \in L^e$$

Services

$$cc_s^{c,c'} = \epsilon_s^{c,c'} + \delta\epsilon_s^{c,c'} \qquad \forall s \in SC, c, c' \in C$$

$$y_{s_i}^c = y_{s_f}^c \qquad \forall s \in SC, c \in C$$

Shunting

+ flow conservation, fleet size, infrastructure capacity, etc.

Case Study: Rolling Stock Assignment

$p_{a,\ell}^{3-4}, p_{a,\ell}^{4-10}$	#C	TSOC	EMOC	PEC	#CC	OI	ST
0.5,3	66	78331.44	1635.76	3019	16	43.35	13.05
1,5	64	80099.76	1265.04	3554	20	42.30	24.56
3,6	65	82896.24	1281.36	4403	18	40.73	8.55

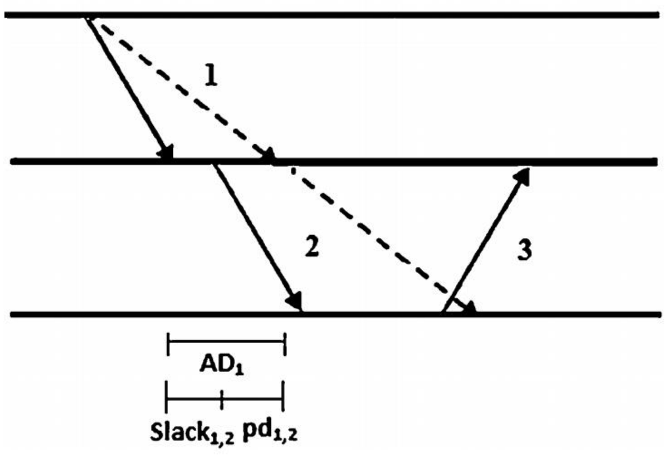
Case	TSOC	EMOC	#EMRH	PEC	#CC	#CCRH	OI	ST
NoRob	80099.76	1265.04	13	3554	20	4	42.30	24.56
Rob	80413.68	1265.04	10	3248	20	3	42.11	28.54

Outline

Train Routing

Sequences for each train unit in the train network once the RS assignment for each service is known.

Robustness: minimise expected delay in train connections.



Train Routing: Model Formulation

$$\min\ E\left[\sum_{i\in I}\sum_{j\in I} pd_{i,j}x_{i,j}\right] + \sum_{i\in I}\sum_{j\in I} cr_{i,j}x_{i,j}$$

Objective function

$$\sum_{j\in I}\ \sum_{\substack{t_i,t_j\in T\\ t_j\geq t_i+\gamma_{i,j}}} co_{i,j}x_{i,j} = \alpha_i \quad \forall i\in I$$

Sequencing

$$\sum_{i\in I}\ \sum_{\substack{t_i,t_j\in T\\ t_j\geq t_i+\gamma_{i,j}}} co_{i,j}x_{i,j} = \beta_j \quad \forall j\in I$$

$$x_{i,j}\in\{0,1\} \qquad \forall i,j\in I$$

Variables

Case Study: Train Routing

	RRTRM	RENFEP	Expected Delay Reduction
Line C5	111.825	141.373	20.91 %

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Crew Scheduling

CREW PLANNING

- CREW SCHEDULING
- CREW ROSTERING

Assign crew resources to certain tasks derived from the schedule planning problem.

- **Task/Operation:** shortest single operation between two stations (e.g., train services, empty services, composition changes).

- **Duty:** sequence of tasks/operations to be performed by a single crew member withing a working shift.

Crew Scheduling: Model Formulation

$$\min \sum_{i \in M} \sum_{j \in M} \sum_{d \in D} a^{i,j} e^{i,j} \epsilon^{d}_{i,j} + \sum_{i \in M} b^{i} (1 - \sum_{d \in D} \alpha^{d}_{i})$$ Objective function

$$\sum_{d \in D} \alpha^{d}_{i} \leq 1 \qquad \forall i \in M$$ Tasks

$$\alpha^{d}_{i} + \delta^{d}_{i} \leq 1 \qquad \forall i \in M; d \in D$$ Deadheading

$$\sum_{i \in M} \sum_{j \in M} e^{i,j} \epsilon^{d}_{i,j} \leq 1 \qquad \forall d \in D$$ Duties

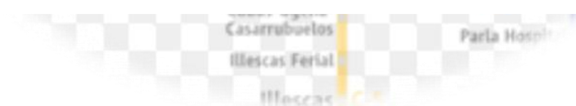
$$\sum_{i \in M} c^{i,j} \gamma^{d}_{i,j} + \sum_{i \in M} d^{i,j} \beta^{d}_{i,j} + \sum_{k \in M} e^{j,k} \epsilon^{d}_{j,k} = \alpha^{d}_{j} + \delta^{d}_{j} \quad \forall j \in M, d \in D$$ Sequences

$$\sum_{i \in M} \sum_{j \in M} \beta^{d}_{i,j} \cdot et_{i} - \sum_{i \in M} \sum_{j \in M} \epsilon^{d}_{i,j} \cdot st_{i} \leq w_{max} \qquad \forall d \in D$$ Breaks

+ other.

Case Study: Crew Scheduling

Time	Duties	Uncovered tasks	Efficiency	Objective function
89	0	369	19.56	699.35
191	48	106	24.19	252.45
279	54	73	32.65	196.19
968	93	5	54.76	105.41
1227	82	6	56.63	94.88
3727	75	1	64.42	77.85
86400	66	1	70.97	68.85



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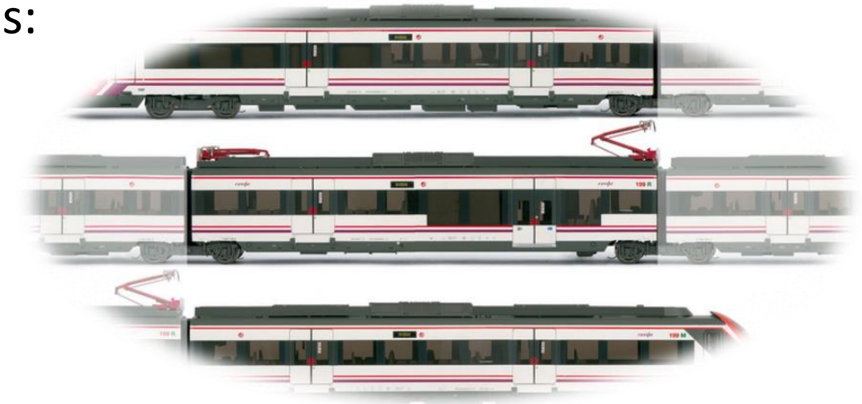
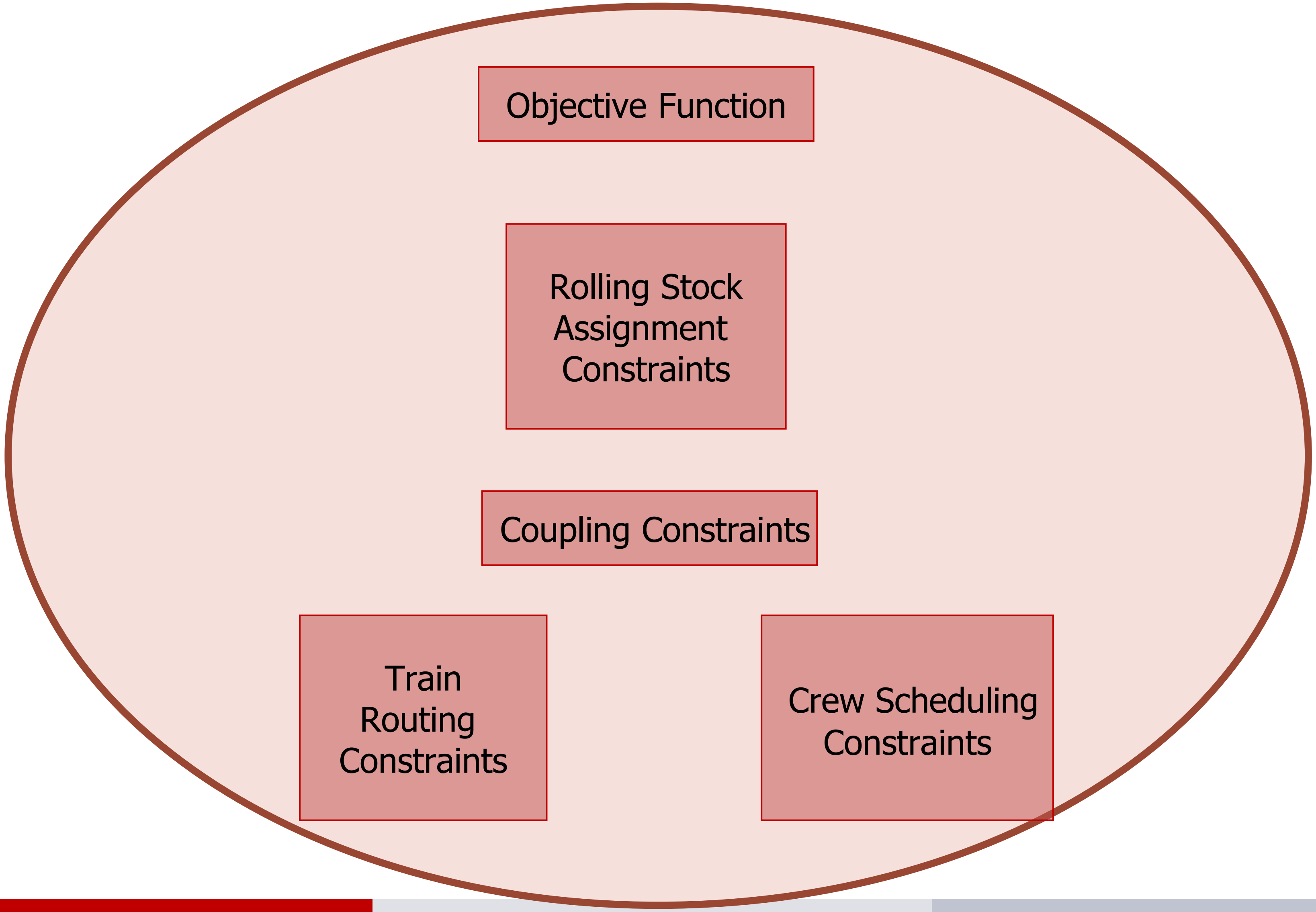
Optimization Model

- We are **integrating** several sub-problems:
  - Rolling stock assignment;
  - Train routing;
  - And crew scheduling.



Optimisation Model



Model Formulation

Objective function

$$\min z = \sum_{\ell \in L} \sum_{c \in C} oc_c km_\ell x_\ell^c + \sum_{\ell \in L^t} \sum_{a \in A_\ell} upc_{a,\ell} up_{a,\ell} + \sum_{s \in SC} \sum_{c,c' \in C} \vartheta_s \cdot cc_s^{c,c'} +$$
$$\sum_{i,j \in I} \sum_{s,s' \in S} \sum_{c \in C} \left[\psi \cdot E \left[pd_{i,s}^{j,s'} \right] + \zeta \cdot cr_{i,s}^{j,s'} \right] \cdot seq_{i,s}^{j,s',c} +$$
$$\sum_{i \in I} \sum_{s \in S} \sum_{j \in I} \sum_{s' \in EAS_s} \sum_{d \in D} e_{i,s,j,s'}^d \cdot \varepsilon_{i,s,j,s'}^d + \sum_{i \in I} \sum_{s \in S} \sum_{s' \in TAS_s} \left[a_{i,s,s'} (1 - \sum_{d \in D} \chi_{i,s,s'}^d) \right]$$

Model Formulation

Constraints

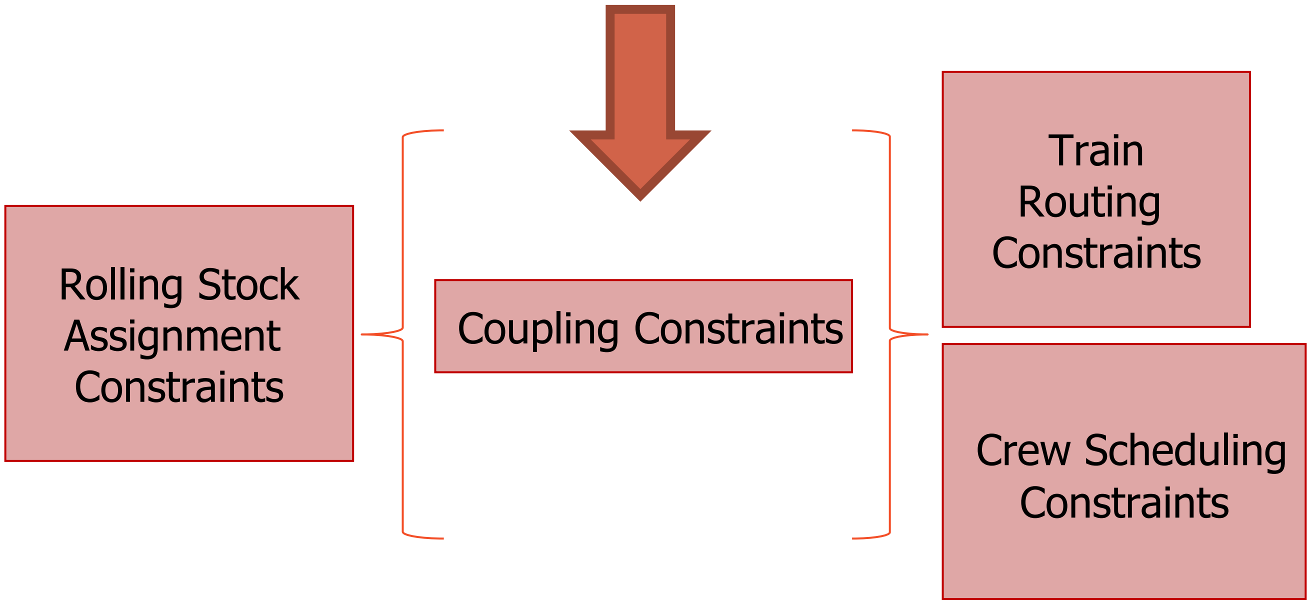
- **Passenger demand** is linked to the capacity of the allocated train units;
- as for the **rolling stock**, each service gets at most one composition; flow conservation is ensured; fleet size; capacity of the stations is controlled;
- **coupling constraints** establish the relationship between rolling stock assignment, train routing and crew scheduling constraints;
- for **train routing**, each operation gets a predecessor and a successor operation (sequence constraints).
- and for **the crew scheduling**, every constructed duty is feasible and legal with respect labour rules.

Model Formulation

Coupling Constraints

$$x_\ell^c = \varphi_{1,s}^c \qquad \forall \ell \in L, c \in C, s \in SC : \alpha_{\ell,s} = -1$$

$$x_\ell^c = \phi_{1,s}^c \qquad \forall \ell \in L, c \in C, s \in SC : \alpha_{\ell,s} = 1$$



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Solution Approach

- The problem has three decision levels:
    1. first, the rolling stock assignment and shunting,
    2. second on the train routing,
    3. and third on the crew scheduling.

- To apply Benders decomposition:
    - a master problem, feasible rolling stock and shunting schedules,
    - a sub-problem which gives the train routing for each schedule,
    - and another sub-problem which provides crew schedules.

Solution Approach: Benders Train Routing Sub-model

$$\min z = \left[\sum_{i,j \in I} \sum_{s,s' \in S} \sum_{c \in C} \left[\psi \cdot E\left[pd_{i,s}^{j,s'}\right] + \zeta \cdot cr_{i,s}^{j,s'} \right] \cdot seq_{i,s}^{j,s',c} \right]$$

$$\sum_{j \in I} \sum_{s' \in S} seq_{i,s}^{j,s',c} = \tilde{\alpha}_{i,c} \phi_{i,s}^{c,it} \quad \left(\kappa_{i,s}^{c}\right) \quad \forall i \in I,\ s \in S,\ c \in C$$

$$\sum_{i \in I} \sum_{s' \in S} seq_{i,s}^{j,s',c} = \tilde{\beta}_{j,c} \varphi_{j,s}^{c,it} \quad \left(\lambda_{j,s}^{c}\right) \quad \forall j \in I,\ s \in S,\ c \in C$$

$$seq_{i,s}^{j,s',c} \in \{0,1\}$$

$$\min z = \left[\sum_{i' \in I'} \sum_{j' \in I'} \left(\psi E\left[pd_{i'}^{j'}\right] + \zeta cr_{i'}^{j'} \right) seq_{i'}^{j'} \right]$$

Subject to :

$$\sum_{j' \in CO_{i'}^{j'}} seq_{i'}^{j'} = \alpha_{i'}^{it} \quad (\kappa_{i'}) \quad \forall i' \in I'$$

$$\sum_{i' \in CO_{i'}^{j'}} seq_{i'}^{j'} = \beta_{j'}^{it} \quad (\lambda_{j'}) \quad \forall j' \in I'$$

$$seq_{i'}^{j'} \in \mathcal{R}^{+} \quad \forall i', j' \in I'$$

Solution Approach: Benders Crew Scheduling Sub-model

$$\min z = \sum_{i\in I}\sum_{s\in S}\sum_{j\in I}\sum_{s'\in EAS_s}\sum_{d\in D} e^d_{i,s,j,s'}\cdot\varepsilon^d_{i,s,j,s'} + \sum_{i\in I}\sum_{s\in S}\sum_{s'\in TAS_s}\left[a_{i,s,s'}\left(1-\sum_{d\in D}\chi^d_{i,s,s'}\right)\right]$$

$$\sum_{d\in D}\chi^d_{i,s,s'} \le \sum_{c\in C}\frac{1}{2}\left(\phi^{c,it}_{i,s'}+\varphi^{c,it}_{i,s}\right) \qquad (\eta_{i,s,s'})\forall i\in I; s\in S; s'\in TAS_s$$

$$\chi^d_{i,s,s'}+\varpi^d_{i,s,s'} \le \sum_{c\in C}\frac{1}{2}\left(\phi^{c,it}_{i,s'}+\varphi^{c,it}_{i,s}\right) \qquad (\iota^d_{i,s,s'})\forall i\in I; s\in S, s'\in TAS_s, d\in D$$

+ other constraints

$$\min z = \sum_{i'\in I'}\sum_{j'\in I'}\sum_{d\in D} e^d_{i',j'}\cdot\varepsilon^d_{i',j'} + \sum_{i'\in I'}\left[a_{i'}\left(1-\sum_{d\in D}\chi^d_{i'}\right)\right]$$

$$\sum_{d\in D}\chi^d_{i'} \le \tau^{it}_{i'} \qquad (\eta_{i'})\forall i'\in I'$$

$$\chi^d_{i'}+\varpi^d_{i'} \le \tau^{it}_{i'} \qquad (\iota^d_{i'})\forall i'\in I'; d\in D$$

+ other constraints

Solution Approach: Benders Master Model

$$\min z = \sum_{\ell \in L} \sum_{c \in C} oc_c km_\ell x_\ell^c + \sum_{\ell \in L^t} \sum_{a \in A_\ell} upc_{a,\ell} up_{a,\ell} + \sum_{s \in SC} \sum_{c,c' \in C} \vartheta_s \cdot cc_s^{c,c'} + \omega + \xi$$

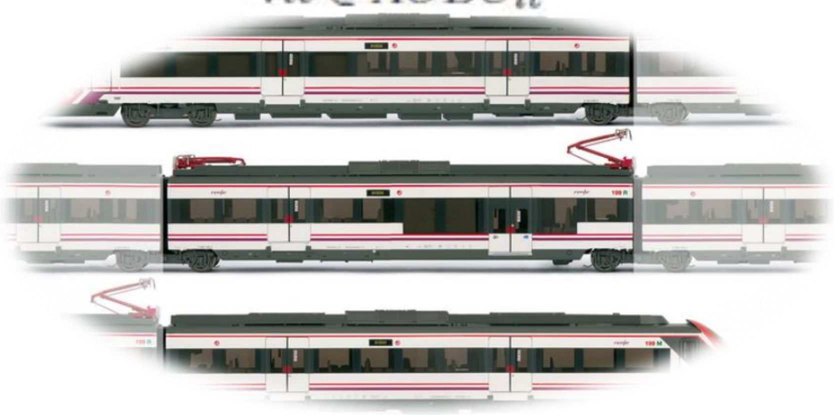
$$\omega \geq \sum_{i \in I} \sum_{s \in S} \sum_{c \in C} \tilde{\alpha}_{i,c} \phi_{i,s}^c \kappa_{i,s}^{c,it} + \sum_{j \in I} \sum_{s \in S} \sum_{c \in C} \tilde{\beta}_{j,c} \varphi_{j,s}^c \lambda_{j,s}^{c,it} \qquad \forall it \in AOBC_{it}$$

$$\xi \geq \sum_{i \in I} \sum_{s \in S} \sum_{s' \in TAS_s} \sum_{c \in C} \frac{1}{2} (\phi_{i,s'}^{c,it} + \varphi_{i,s}^{c,it}) \eta_{i,s,s'}^{it} +$$

$$\sum_{i \in I} \sum_{s \in S} \sum_{s' \in TAS_s} \sum_{d \in D} \sum_{c \in C} \frac{1}{2} (\phi_{i,s'}^{c,it} + \varphi_{i,s}^{c,it}) \iota_{i,s,s'}^{d,it} +$$

$$\sum_{d \in D} \nu_d + w_{max} (\varsigma_d + \upsilon_d) \qquad \forall it \in AOBC_{it}$$

+ rolling stock assignment constraints.



Outline

Case Study

- Rapid transit network in Madrid.
- Line C5: 22 stations and 4 depots.
- 325 scheduled services.
- More than 430 tasks/operations.

Case Study

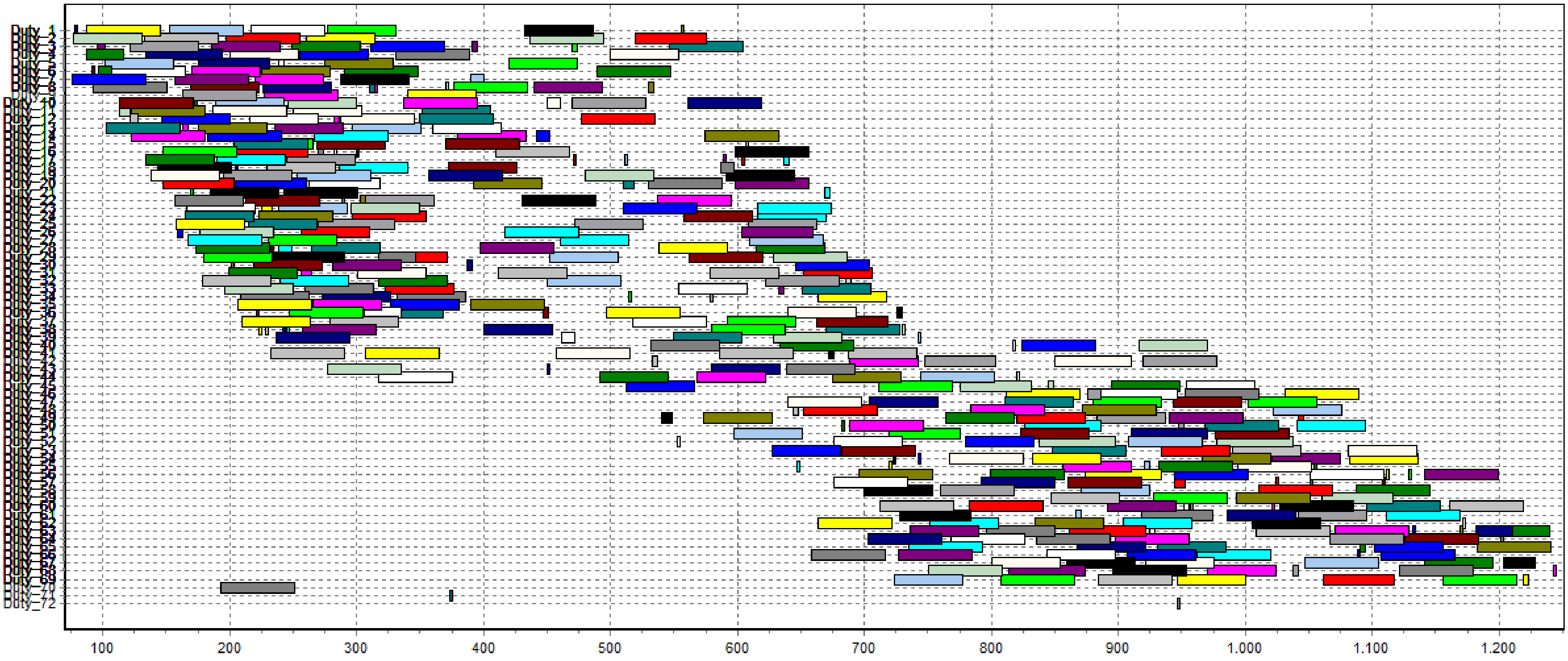
- We compare four approaches:
  - ✓ The current solution;
  - ✓ Sequentially obtained solutions;
  - ✓ Partially integrated solutions;
  - ✓ And integrated solutions.

Case Study

Item \ Approach	RS+TR+CS	RS+TR and CS	RS, TR and CS	Current
#C	67	66	64	74
TSOC	80959.44	80413.68	80099.70	109765.20
ESOC	1305.92	1457.44	1265	2232.10
UPC	3248	3554	3554	874
#CC	14	14	20	0
EDP (min.)	74.19	61.03	111.80	141.37
EDR (%)	47.52	56.82	20.91	-
#D	68	69(+3)	67(+3)	66
EFF (%)	67.77	66.76(63.98)	68.01(65.10)	68.85
CSC	13723.22	15125.49	14731.76	13347.45
TOC	95988.58	96996.61	96096.46	125344.75

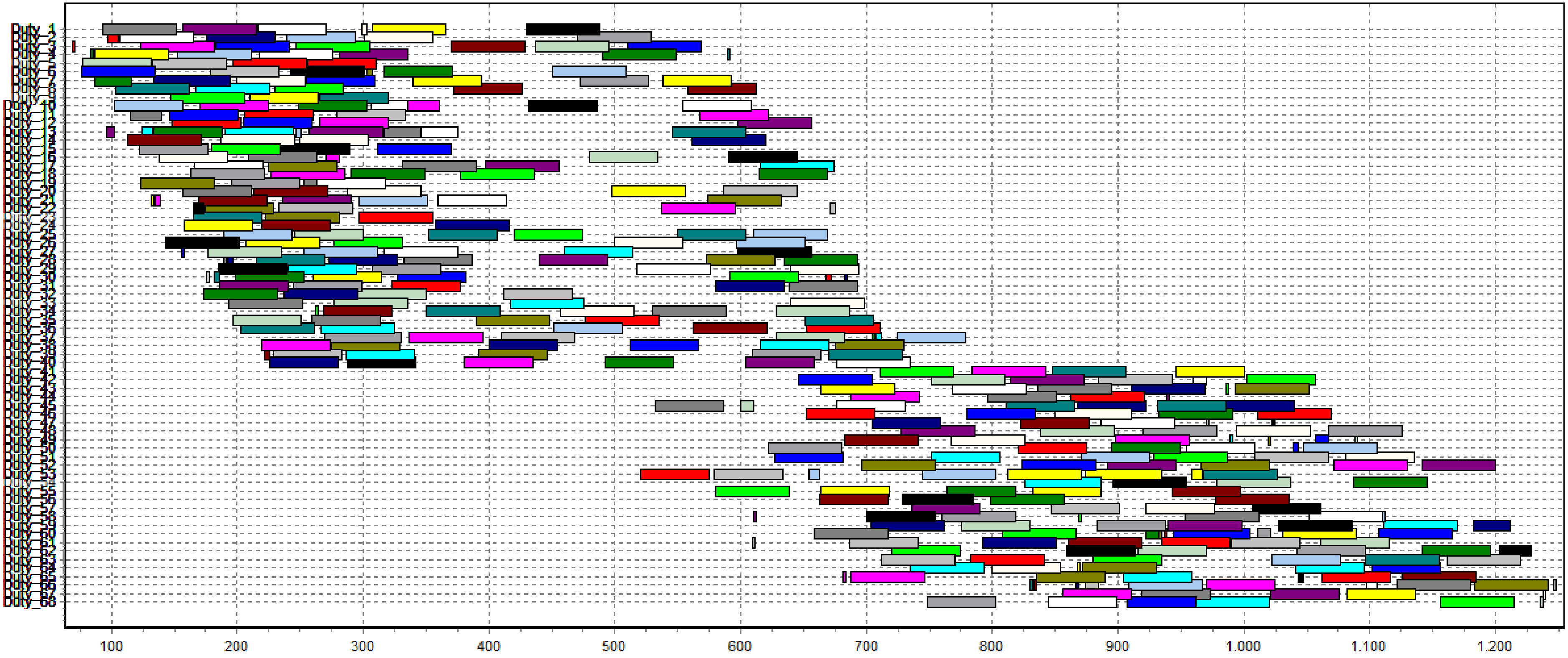
Case Study

Sequential approach: Crew Scheduling solution



Case Study

Integrated approach: Crew Scheduling solution



Conclusions

- We have presented **an integrated approach** to the problems of rolling stock assignment, train routing and crew scheduling.
- To the best of our knowledge this is the **first attempt** in providing operational plans in such an integrated way.
- Because the resulting mathematical model is huge, we use a Benders decomposition based **heuristic** to solve the integrated approach.
- Computational experiments, drawn from **realistic cases**, show that the integrated approach outperforms all the rest of the approaches tested.

THANK YOU

ANY QUESTIONS?