A new mixed integer linear programming formulation for a maintenance problem in Italian railways

SAPIENZA
UNIVERSITÀ DI ROMA

Dipartimento di Scienze Statistiche
Paolo Dell'Olmo
E-mail: paolo.dellolmo@uniroma1.it

Lavinia Amorosi – PhD Sapienza
Gianni Luca Giacco - Trenitalia

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High Speed Trains

Rail transport increased its market through the introduction of high-speed trains and it overcame, on small and medium distances, the road and air transport. The growing demand for high-speed trains and the competition with other operators lead companies to **increase rail transport services to better meet the demand** with obvious implications on the company's profit.

Maintenance

From an economic point of view the importance of the optimization of maintenance processes can be easily understood if we consider that for **high-speed trains**, most of **30% of the cost of the life cycle is due to maintenance operations**. The high-speed trains have to undergo special maintenance services, with predetermined **constraints ill-structured** and difficult to represent mathematically. In addition, the maintenance service is highly specialized and can only run on **specific platforms in the stations** of destination and is subject to a more highly articulated process.

Literature References

The problem of routing of trains with that of the maintenance have been addressed and faced by different authors.

- G. Maroti, L.G Kroon, Maintenance routing for train units: the transition model. *Transportation Science,* (2005).

- G. Budai, D. Huisman, R. Dekker, Scheduling Preventive Railway Maintenance Activities. *Journal of the Operational Research Society*, (2006).

- G.L Giacco, A. Dariano, D. Pacciarelli, Rolling Stock Rostering Optimization under Maintenance Constraints, *Journal of Intelligent Transportation Systems*, (2013).

Preliminary Definitions

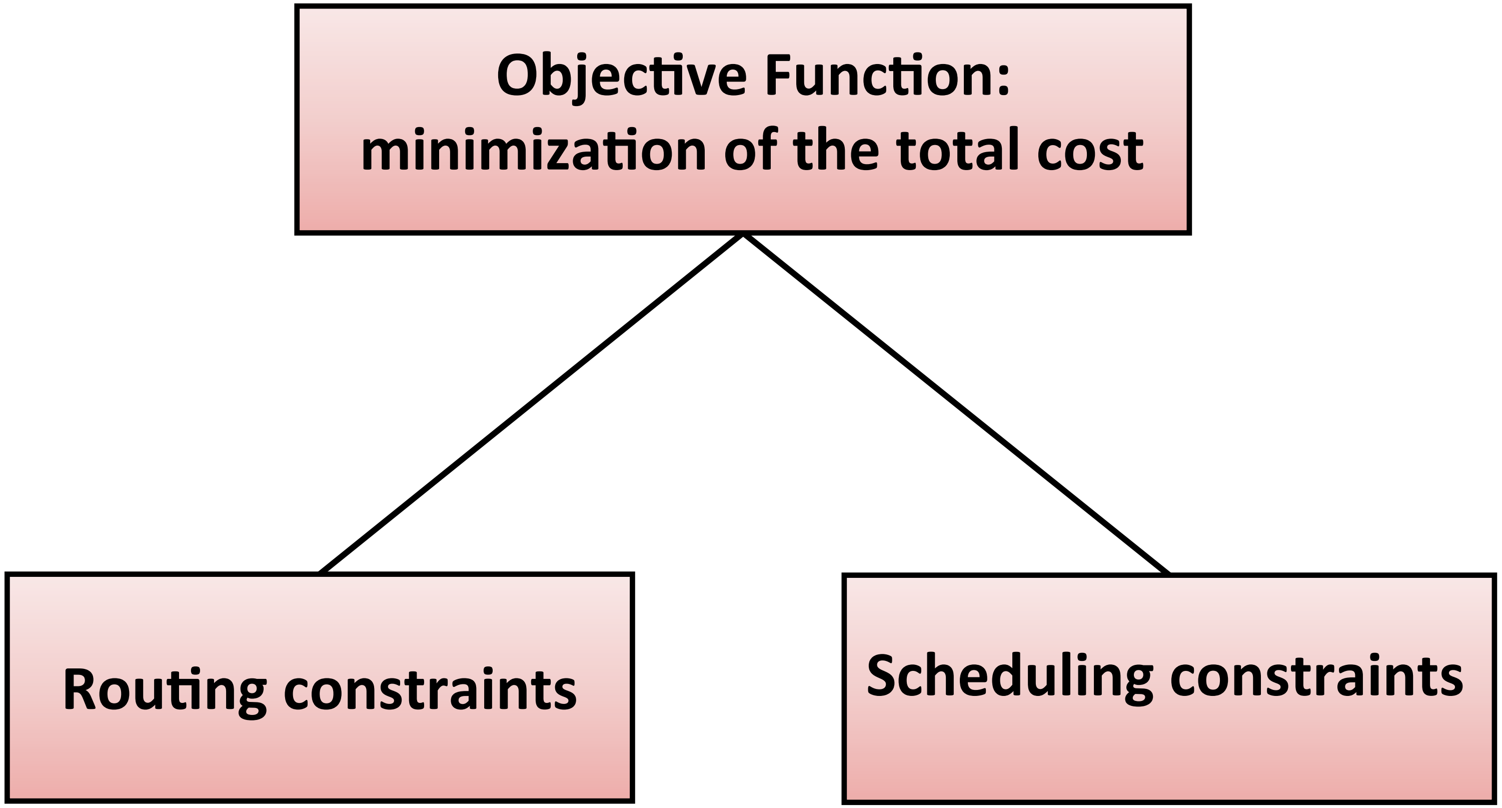
- **Train Service i :** a route from a departure station d_i at departure time t_i^d to an arrival station a_i at arrival time t_a^i that must be covered by a specific train.

- **Roster**: a cycle spanning over several working days that covers all the services and the required maintenance tasks.

- **Network plan**: a document that describes the main train services to be provided.

- **Timetable**: a detailed network plan showing also information on departure and arrival times and the days in which trains will be provided.

The Integrated Approach



Mathematical Model

Notation (1/3)

- V = set of **macro-services**
- U = set of **units** in asset
- L = set of **maintenance levels**

GRAPH: $G=(N, A)$

- $N = U \cup V$, set of nodes (macro services and units)
- A = set of arcs linking time feasible macro-services and asset units to macro services

Notation (2/3)

Decision Variables

$$\forall i \in V \quad \forall j \in V : (i, j) \in A$$

$$x_{i,j} = \begin{cases} 1 \text{ if the macro-service } j \text{ follows the macro-service } i \\ 0 \text{ otherwise} \end{cases}$$

$$\forall i \in U \quad \forall j \in V : (i, j) \in A$$

$$x_{i,j} = \begin{cases} 1 \text{ if the asset unit } i \text{ is assigned to the macro-service } j \\ 0 \text{ otherwise} \end{cases}$$

Notation (3/2)

$$\forall j \in V \quad \forall l \in L$$

$$p_j^l = \begin{cases} 1 \text{ if the maintenance level } l \text{ is provided} \\ \quad \text{before macro-service } j \\ 0 \text{ otherwise} \end{cases}$$

$$\forall i \in V \quad \forall l \in L$$

$g_i^l \geq 0$ means the number of cumulated kilometres by train after performing the macro-service i from the last service of level l

$$\forall i \in V \quad \forall l \in L$$

$h_i^l \geq 0$ means the number of cumulated kilometres by train before performing the macro-service i from the last service of level l

Objective Function

$$min\,(\sum_{i \in U\, j \in V:(i,j) \in A} x_{i,j}) * K_1 + \sum_{l \in L} \sum_{j \in V} p_j^l * c_l + (\sum_{j \in V} \sum_{l \in L} \sum_{k \in S_l} n_{j,l,k}) * K_2$$

Cost of the units in asset

Cost of the maintenance operations

Cost of the train movements in the workshop

Path constraints

$$\begin{cases} \sum_{j \in V : (i,j) \in A} x_{i,j} \leq 1 & \forall i \in V \\ \sum_{i \in U \cup V : (i,j) \in A} x_{i,j} = 1 & \forall j \in V \\ \sum_{j \in V : (i,j) \in A} x_{i,j} \leq 1 & \forall i \in U \end{cases}$$

Maintenance constraints

$$
\begin{cases}
g_i^l = Km_i + h_i^l \quad \forall i \in V \; \forall l \in L \\
h_j^l \geq g_i^l - (1 - x_{i,j}) * M - p_j^l * M \quad \forall l \in L \; \forall (i,j) \in A : \sum_{k \in O_l} d_k \leq t_{i,j} \\
h_j^l \leq g_i^l + (1 - x_{i,j}) * M + p_j^l * M \quad \forall l \in L \; \forall (i,j) \in A : \sum_{k \in O_l} d_k \leq t_{i,j} \\
\sum_{l \in L} p_j^l \leq 1 \quad \forall j \in V \\
h_i^l \leq (1 - p_i^l) * (\gamma_l - Km_i) \quad \forall i \in V \; \forall l \in L \\
g_i^l = Res_{i,l} \quad \forall i \in U \; \forall l \in L
\end{cases}
$$

Example

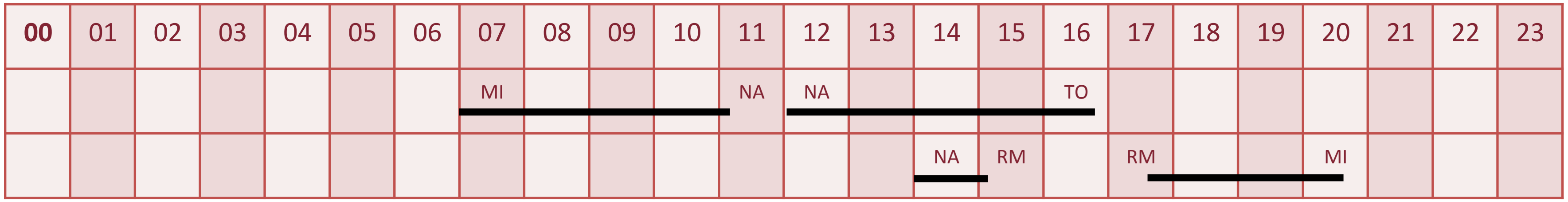
Illustrative example

- Time window: 3 Days
- Units in asset: 3
- Maintenance Levels: 2 (LM1, LM2)
- Deadline LM1: 3700 km
- Deadline LM2: 4500 km
- Duration LM1: 1 h
- Duration LM2: 2 h
- 1 Maintenance workshop placed in Naples

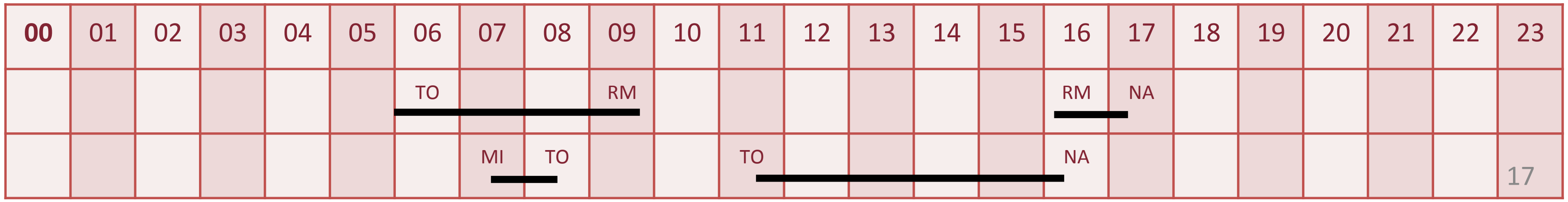
A feasible roster without maintenance

Day 1

Day 2



Day 3



Macro-services

NA-RM + RM-MI + MI-NA = A g1 08:00 – g2 11:05 / km 1580

NA-RM + RM-MI + MI-TO + TO-NA = B g2 14:00 – g3 16:20 / km 1830

NA-MI + MI-NA = C g1 07:00 – g1 16:50 / km 1580

NA-TO + TO-RM + RM-NA = D g2 12:00 – g3 17:20 / km 1830

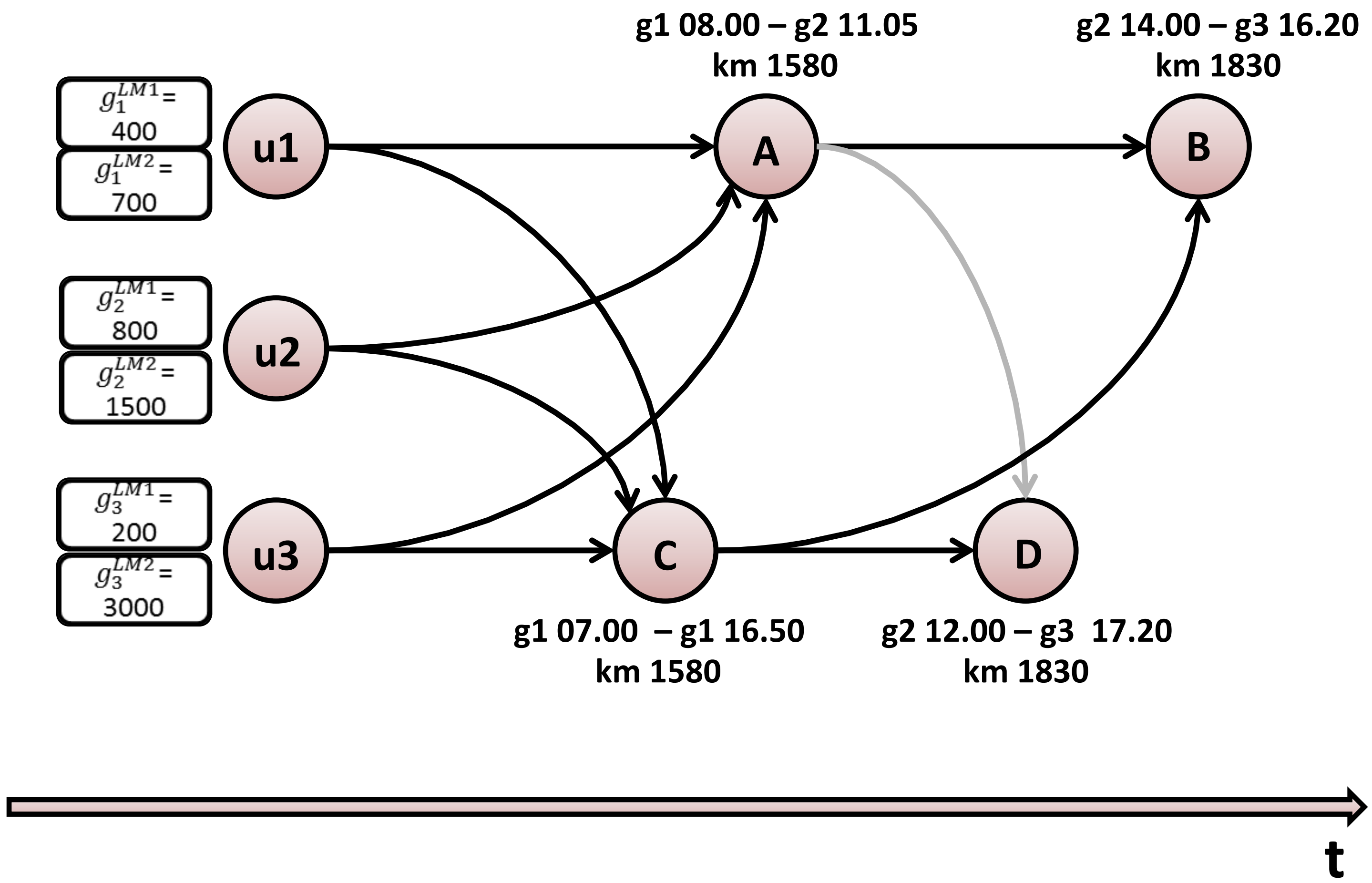
Available Units

g_1^{LM1}= 400
g_1^{LM2}= 700
u1

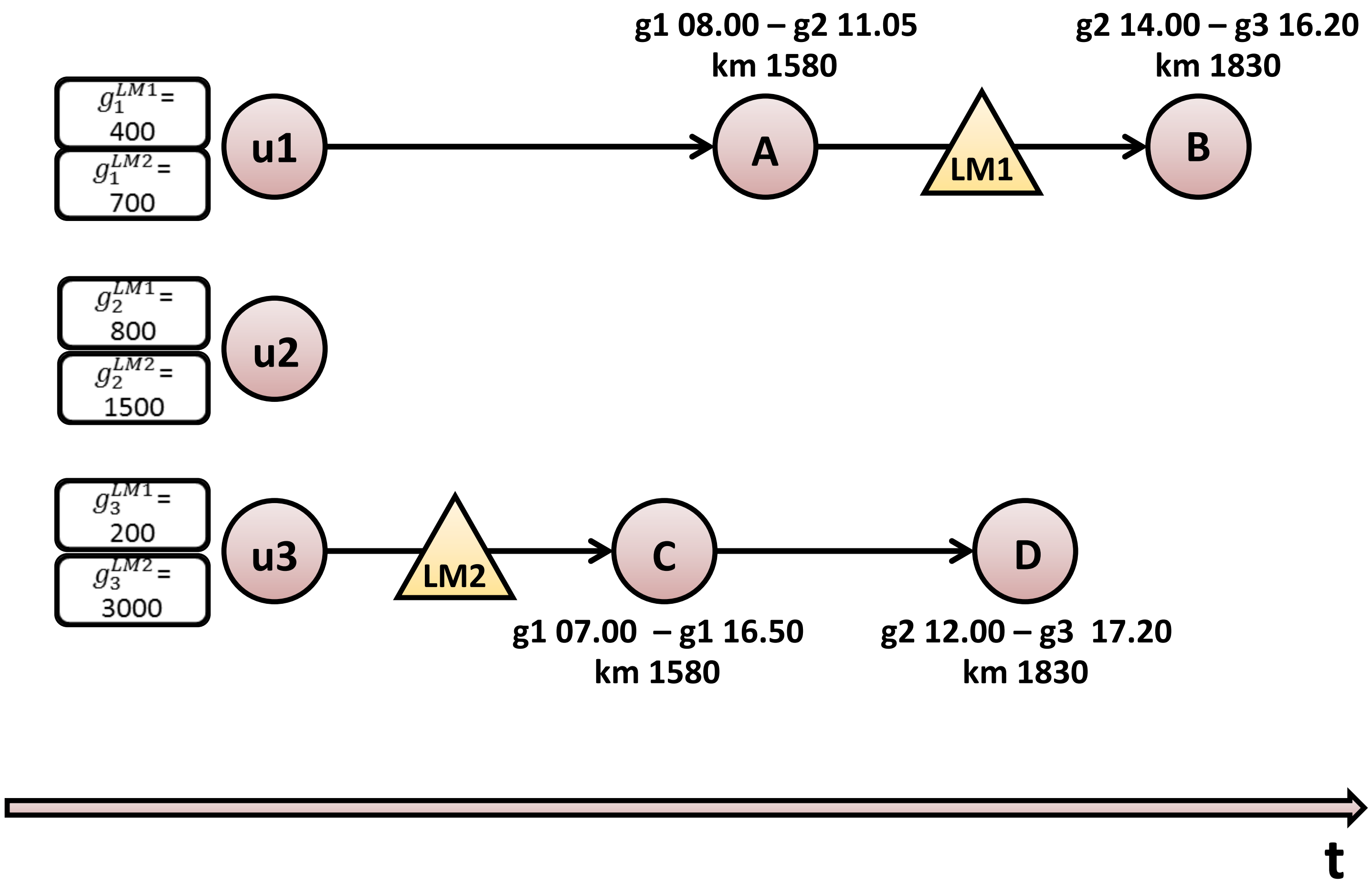
g_2^{LM1}= 800
g_2^{LM2}= 1500
u2

g_3^{LM1}= 200
g_3^{LM2}= 3000
u3

The graph

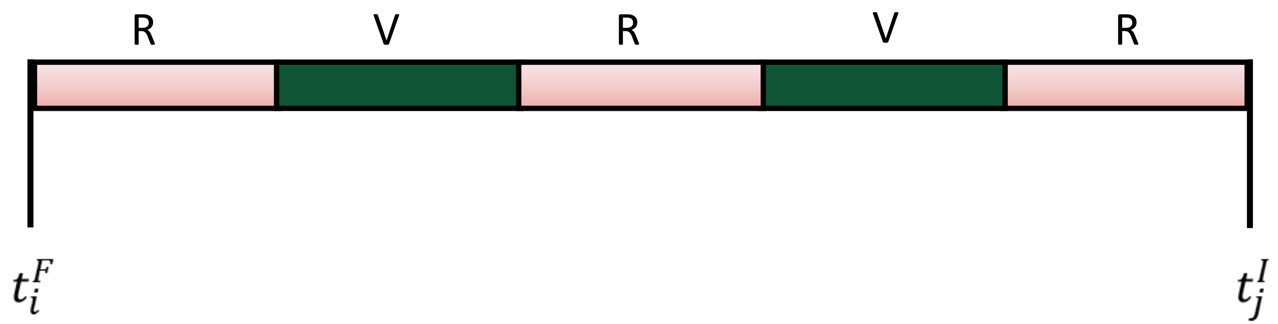

g1 08.00 – g2 11.05
km 1580
g2 14.00 – g3 16.20
km 1830
g_1^{LM1}= 400
g_1^{LM2}= 700
g_2^{LM1}= 800
g_2^{LM2}= 1500
g_3^{LM1}= 200
g_3^{LM2}= 3000
u1
u2
u3
A
B
C
D
g1 07.00 – g1 16.50
km 1580
g2 12.00 – g3 17.20
km 1830
t

A feasible solution



Scheduling Constraints

Number of train movements



for $i = 2, ..., T_{l,k} \ \forall k \in S_l \ \forall l \in L \ \forall j \in V \ \forall r \in R$

$$y^r_{v^{l,k,j}_{i-1,i}} = \begin{cases} 1 \text{ if the virtual task between } i\text{-1-th task and } i\text{-th} \\ \quad \text{task of the sequence } k \text{ for the maintenance level} \\ \quad l \text{ before macro-service } j \text{ is assigned to resource } r \\ 0 \text{ otherwise} \end{cases}$$

Number of train movements

$$\forall k \in S_l \, \forall l \in L \, \forall j \in V \, \forall r \in R$$

$$y^{r}_{v_1^{l,k,j}} = \begin{cases} 1 \text{ if the virtual task before the beginning of the} \\ \quad \text{sequence } k \text{ for the maintenance level } l \text{ before} \\ \quad \text{macro-service } j \text{ is assigned to resource } r \\ 0 \text{ otherwise} \end{cases}$$

$$\forall k \in S_l \, \forall l \in L \, \forall j \in V \, \forall r \in R$$

$$y^{r}_{v_{T_{l,k}}^{l,k,j}} = \begin{cases} 1 \text{ if the virtual task after the end of the sequence } k \\ \quad \text{for the maintenance level } l \text{ before macro-service} \\ \quad j \text{ is assigned to resource } r \\ 0 \text{ otherwise} \end{cases}$$

Number of train movements

for $i = 2, \ldots T_{l,k}\ \forall k \in S_l\ \forall l \in L\ \forall j \in V$

$$w^{l,k,j}_{i-1,i} = \begin{cases} 1 \text{ if there is a movement between } i\text{-1-th task and} \\ \quad i\text{-th task of the sequence } k \text{ for the maintenance} \\ \quad \text{level } l \text{ before macro-service } j \\ 0 \text{ otherwise} \end{cases}$$

for $i = 2, \ldots T_{l,k}\ \forall k \in S_l\ \forall l \in L\ \forall j \in V\ \forall r \in B_{i-1} \cap B_i$

$$s^{l,k,j}_{i-1,i,r} = \begin{cases} 1 \text{ if } i\text{-1-th task and } i\text{-th task of the} \\ \quad \text{sequence } k \text{ for the maintenance level } l \text{ before} \\ \quad \text{macro-service } j \text{ are assigned to resource } r \\ 0 \text{ otherwise} \end{cases}$$

Number of train movements

$$\forall k \in S_l \, \forall l \in L \, \forall j \in V \;\; \text{per } i = 2, ..., T_{l,k}$$

$$\begin{cases} w_{i-1,i}^{l,k,j} \leq 1 - \sum_{r \in R} y_{v_{i-1,i}^{l,k,j}}^{r} \\ w_{i-1,i}^{l,k,j} \leq 1 - \sum_{r \in B_{i-1} \cap B_i} s_{i-1,i,r}^{l,k,j} \\ w_{i-1,i}^{l,k,j} \geq (1 - \sum_{r \in R} y_{v_{i-1,i}^{l,k,j}}^{r}) + (1 - \sum_{r \in B_{i-1} \cap Bi} s_{i-1,i,r}^{l,k,j}) - 1 \end{cases}$$

$$\forall k \in S_l \, \forall l \in L \, \forall j \in V \;\; \text{per } i = 2, ..., T_{l,k} \;\; \forall r \in B_{i-1} \cap B_i$$

$$\begin{cases} s_{i-1,i,r}^{l,k,j} \leq \frac{(y_{i-1,r}^{l,k,j} + y_{i,r}^{l,k,j})}{2} \\ s_{i-1,i,r}^{l,k,j} \geq y_{i-1,r}^{l,k,j} + y_{i,r}^{l,k,j} - 1 \end{cases}$$

Number of train movements

$$n_{j,l,k} = \sum_{r\in R} y^{r}_{v_1^{l,k,j}} + \sum_{r\in R} y^{r}_{v_{T_{l,k}}^{l,k,j}} + \sum_{i=2}^{T_{l,k}} \sum_{r\in R} (2 * y^{r}_{v_{i-1,i}^{l,k,j}}) + \sum_{i=2}^{T_{l,k}} w_{i-1,i}^{l,k,j}$$

Train movements associated with the virtual tasks at the start and/or at the end of the maintenance sequence

Train movements associated with the possible virtual task between two consecutive real tasks

Train movements associated with two consecutive real tasks

Computational Results

At the present stage of the work the model was solved in sequential way: first the part related to the problem of routing and then the one related to the scheduling problem in the maintenance workshop. All tests were performed on **50 random instances** (**15 Macro-services – one week**) on a Windows machine with 4 Intel i7 2.3 GHz and 8 GB of RAM. The problem solver used is CPLEX 12.4.

In the table below we show the minimum, maximum and the average computational time (in seconds) for the two parts.

$Model$	Min	Max	Average
First	0.01	0.33	0.06
Second	0.16	0.75	0.24

Thanks !