MODAL
Mathematical Optimization and Data Analysis Laboratories

Template Based Re-Optimization of Rolling Stock Rotations

Ralf Borndörfer, **Boris Grimm**, Thomas Schlechte, Markus Reuther

Zuse Institute Berlin

CASPT 2015, Rotterdam

22. July. 2015

Cooperation Partner

MODAL

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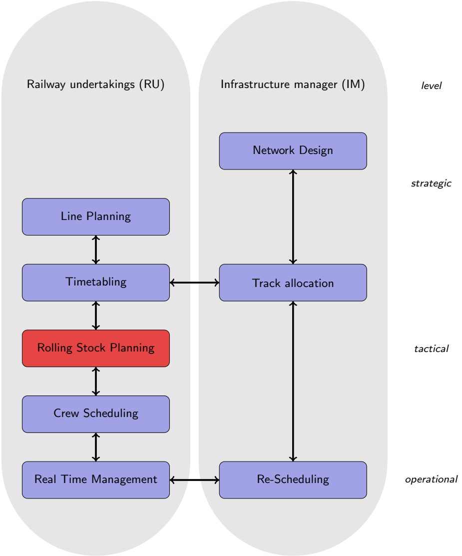
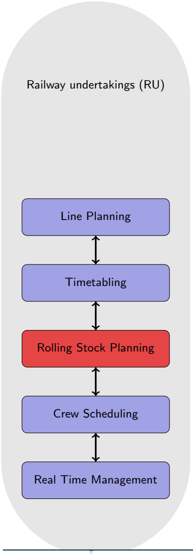
Agenda

What do we do?



Railway undertakings (RU)
Infrastructure manager (IM)
level
Network Design
strategic
Line Planning
Timetabling
Track allocation
Rolling Stock Planning
tactical
Crew Scheduling
Real Time Management
Re-Scheduling
operational

Definition: Rolling Stock Rotation Problem



(Cyclic) Rolling Stock Rotation Problem (RSRP)

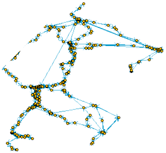
Given:

- Timetable with planned trips
- Set of railway vehicles
- Set of operational rules, e.g., costs for vehicles, limits for couplings

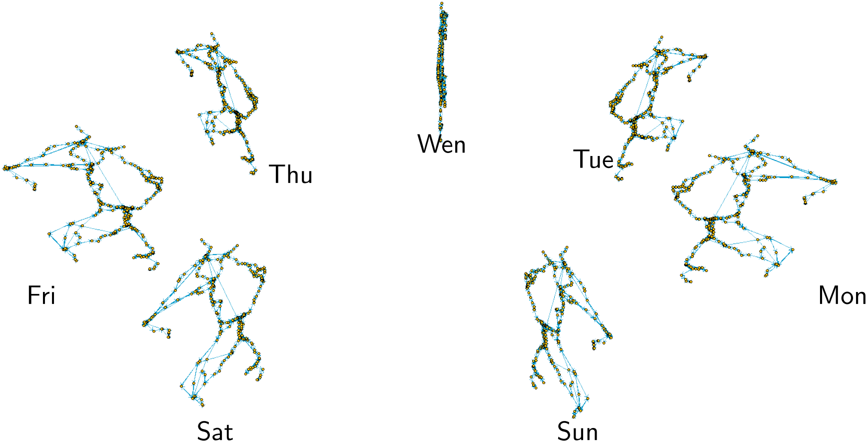
Goal:

- Set of (cyclic) Rotations, i.e., sequences of trips for the vehicles that:
  - Cover all trips
  - Minimize costs / fulfill operational rules

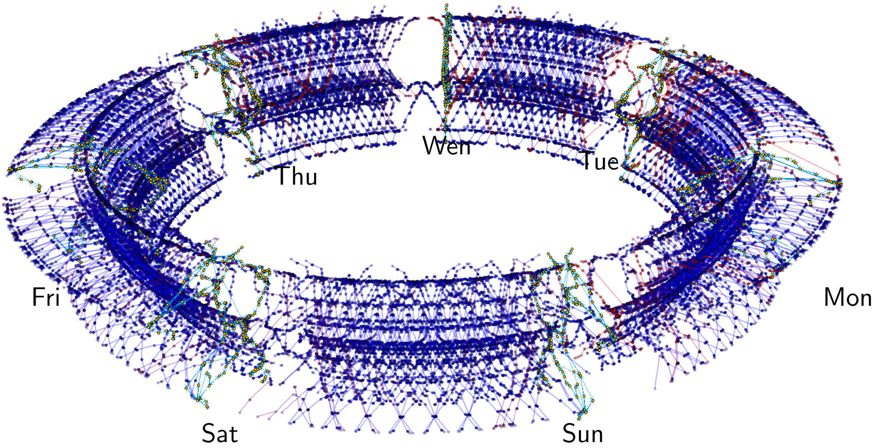
Infrastructure



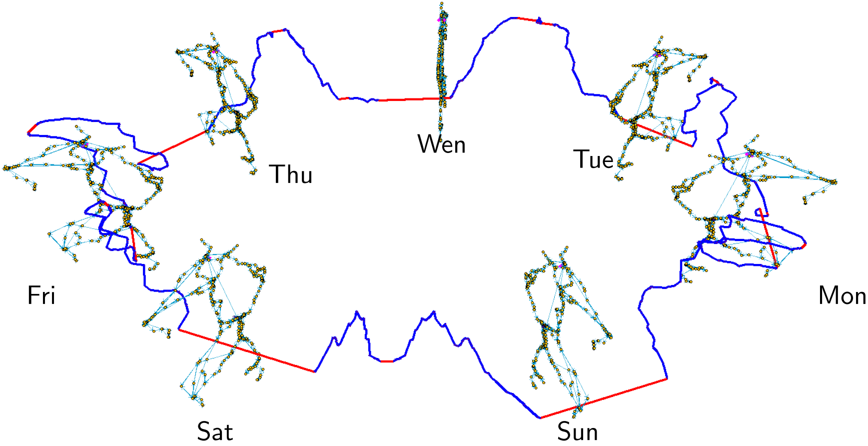
Infrastructure in cyclic standard week



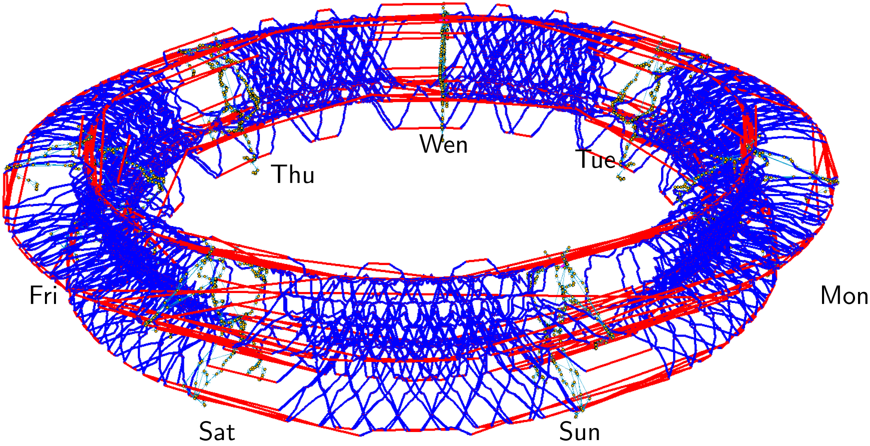
Timetable



Rotation

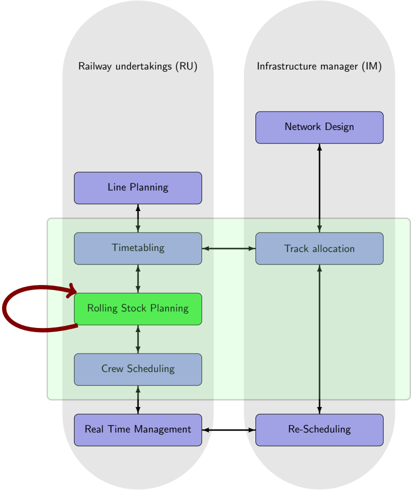


Rotation Plan



Reasons for Re-optimization

- fleet changes



Reasons for Re-optimization

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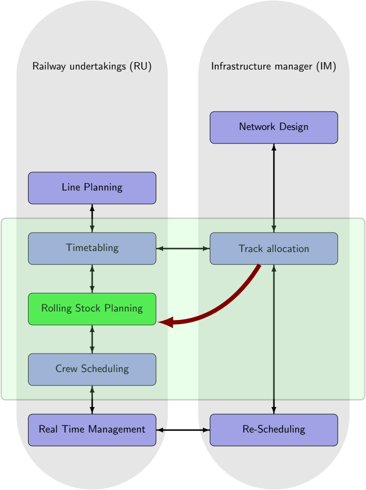

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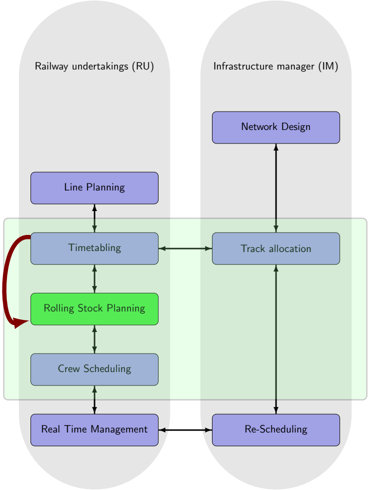
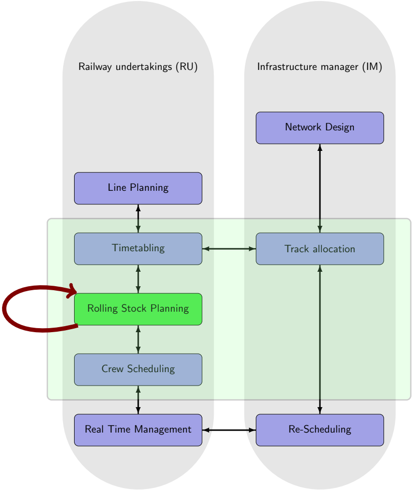


- timetable changes

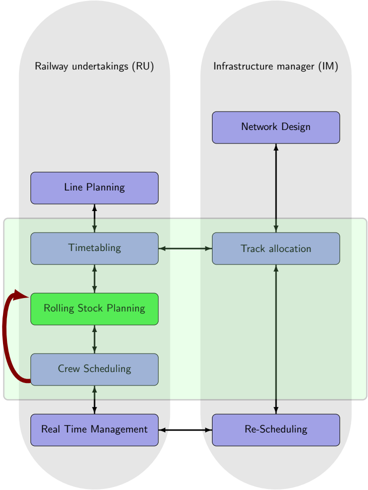
Reasons for Re-optimization

- lockouts of drivers

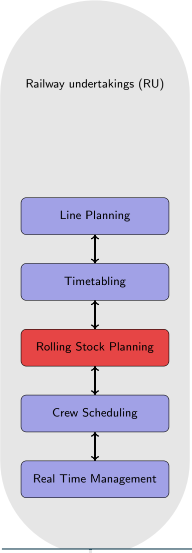


Reasons for Re-optimization

- change of laws



Definition: Re-optimization RSRP



(Cyclic) Re-optimization RSRP (Re-Opt RSRP)

Given:

- Timetable with planned trips
- Set of rotations of railway vehicles
- Set of operational rules, e.g., costs for vehicles, limits for couplings

Goal:

- Set of (cyclic) rotations, i.e., sequences of trips for the vehicles that:
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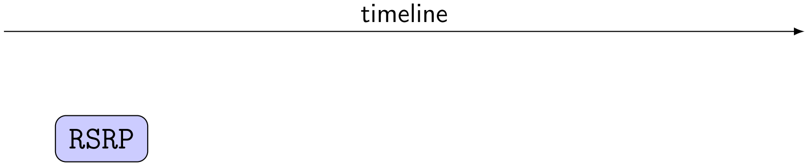
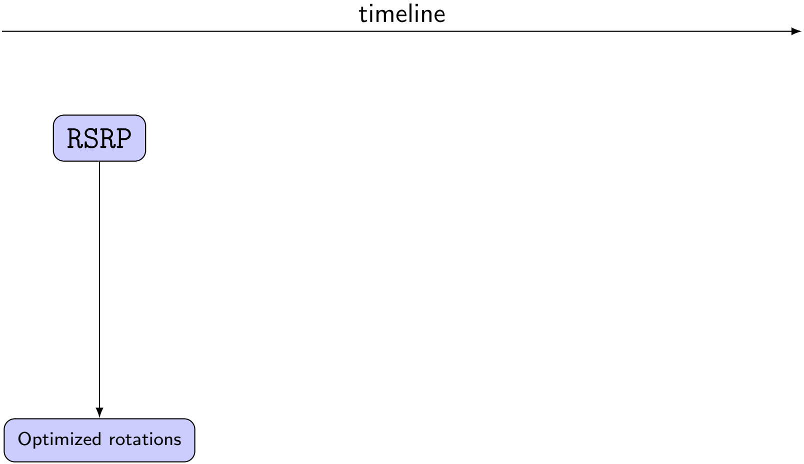
  - Minimize deviations from given rotations
  - Fulfill operational rules

Re-optimization



Re-optimization



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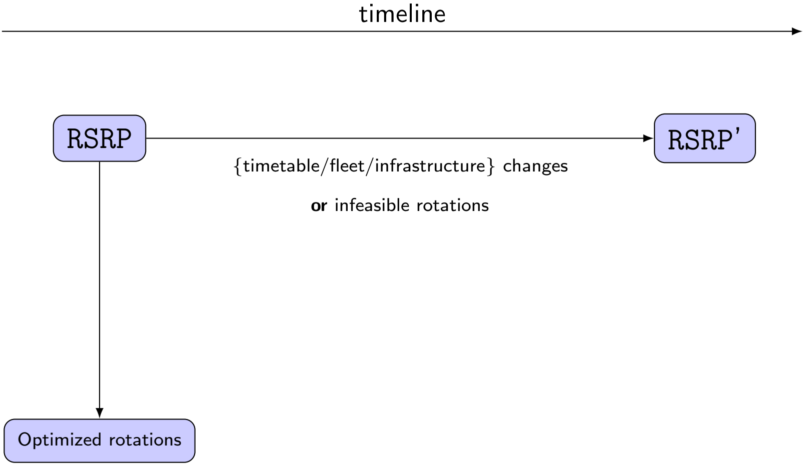
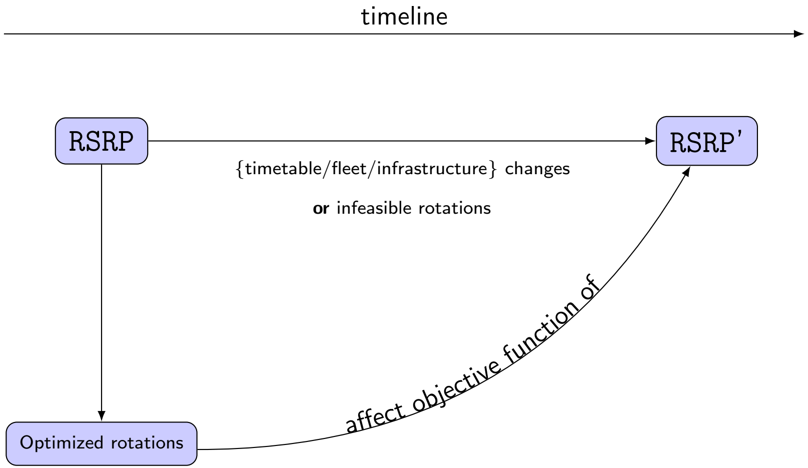
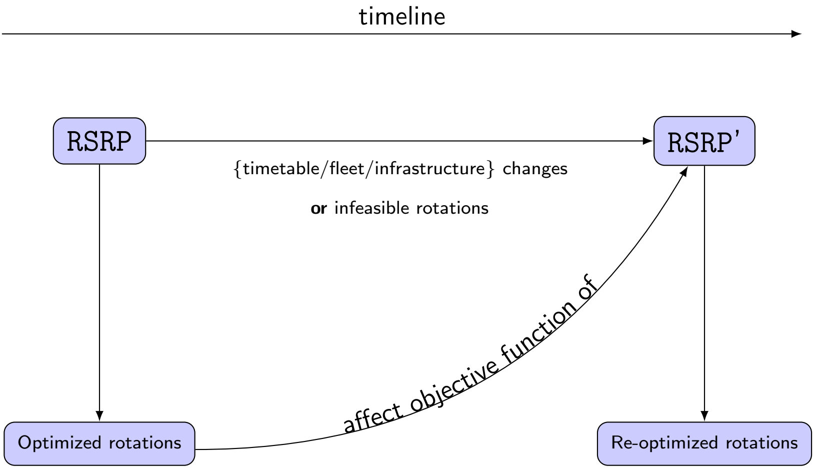


Re-optimization



Re-optimization



Included costs

- optimization cost
  - vehicle cost
  - maintenance cost
  - deadhead distance cost
  - regularity cost
- re-optimization cost (w.r.t. reference solution)
  - cost per deviating vehicle composition (fleets, orientation, position)
  - cost per deviating rotation
  - cost per deviating connection / **deviating deadhhead trip**

Vehicle compositions schould not be changed ...

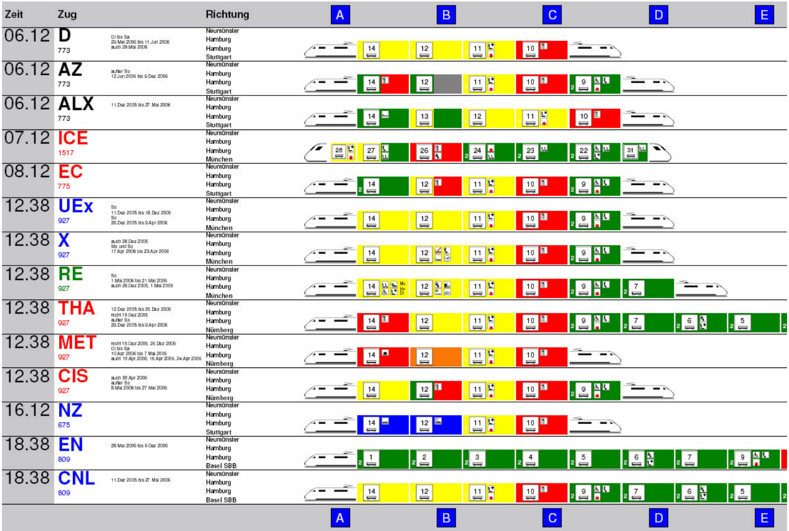


Figure : images/wagenstandanzeiger

Vehicle compositions

Figure : The tree indicates the *driving direction* on the track

Vehicle compositions

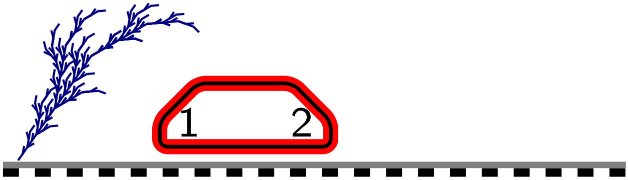


Figure : A red vehicle with an *orientation* (first class in front).

Vehicle compositions

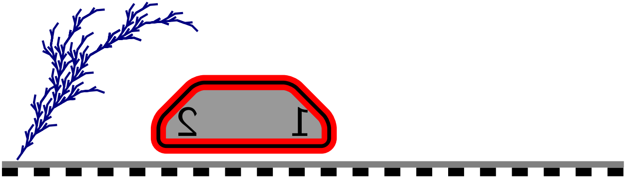


Figure : A red vehicle with the *opposite* orientation (second class in front).

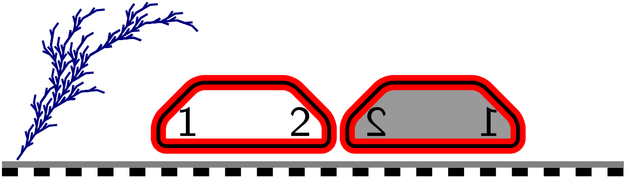


Figure : Two red vehicles with opposite orientations.

Vehicle compositions

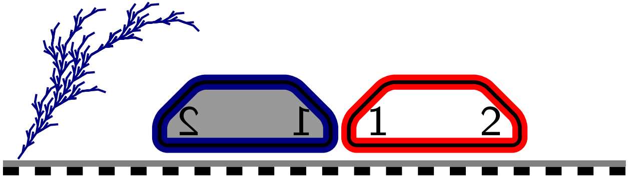


Figure : A red and a blue vehicle with different orientations.

Vehicle compositions

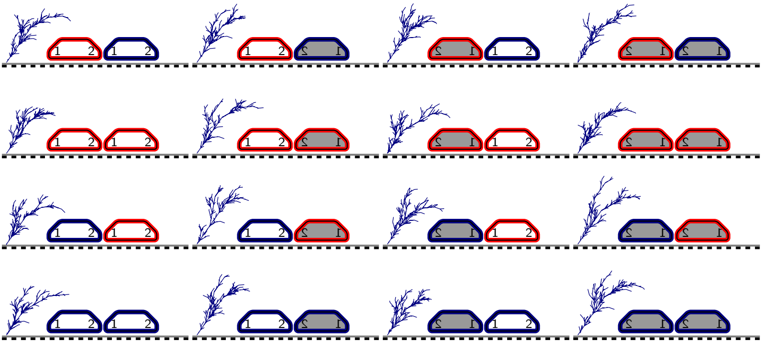


Figure : 16 vehicle compositions for 2 vehicles

The Rolling Stock Rotation Problem (RSRP)

- V is the set of *departures and arrivals of timetabled trips* T

Definition (RSRP hypergraph)

Let (V, A) be a standard directed graph. The RSRP *hypergraph* is denoted by $G = (V, A, H)$ with the set of *hyperarcs* $H \subseteq 2^A$.

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Most important remarks:

- A hyperarc is just a set of standard arcs (in this context).
- A subset of H (not A) is a solution (to be defined).

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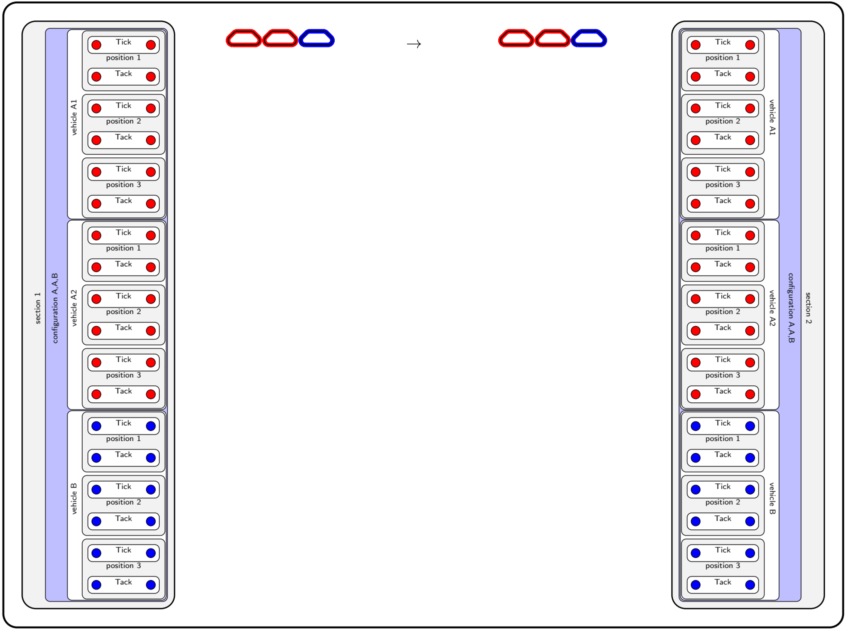
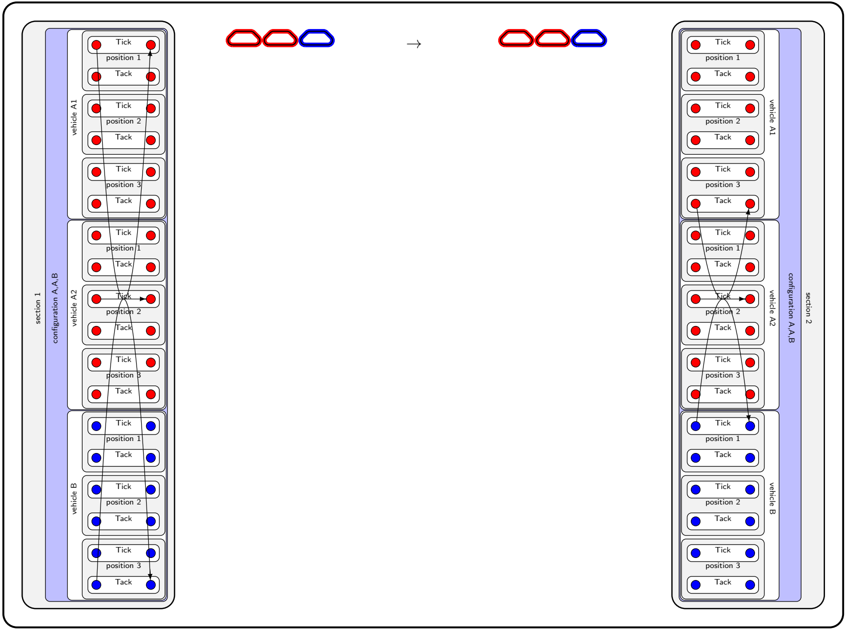

section 1
configuration A,A,B
vehicle A1
vehicle A2
vehicle B
Tick
Tack
position 1
position 2
position 3
→
section 2



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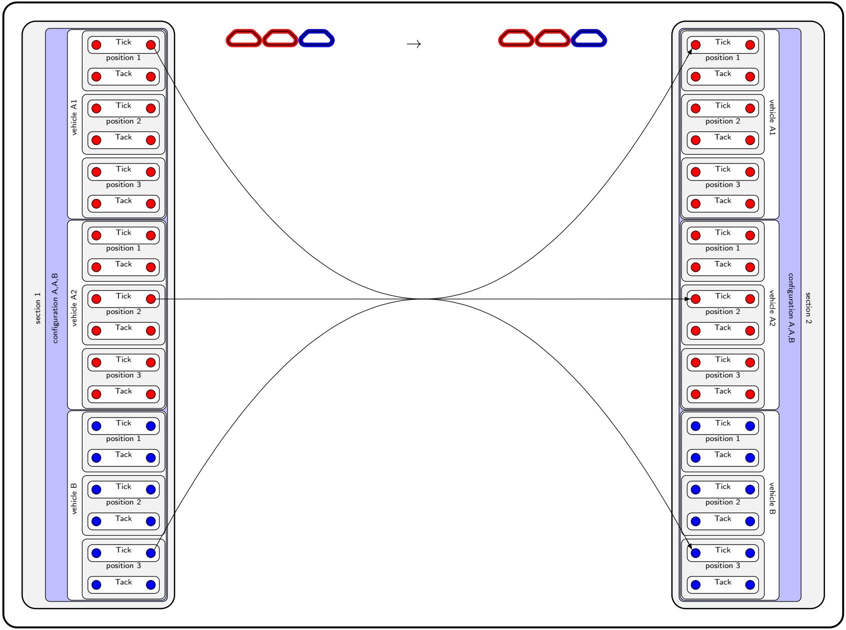

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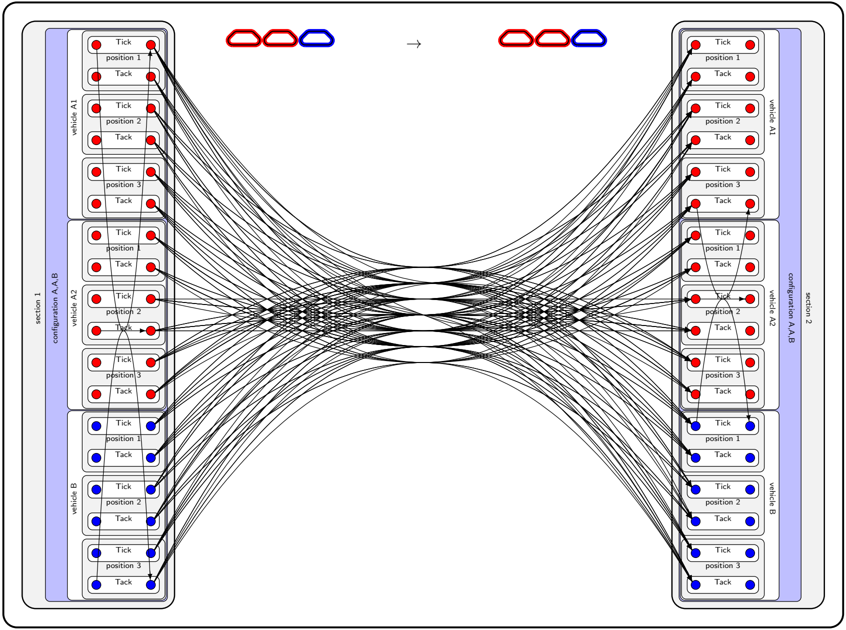

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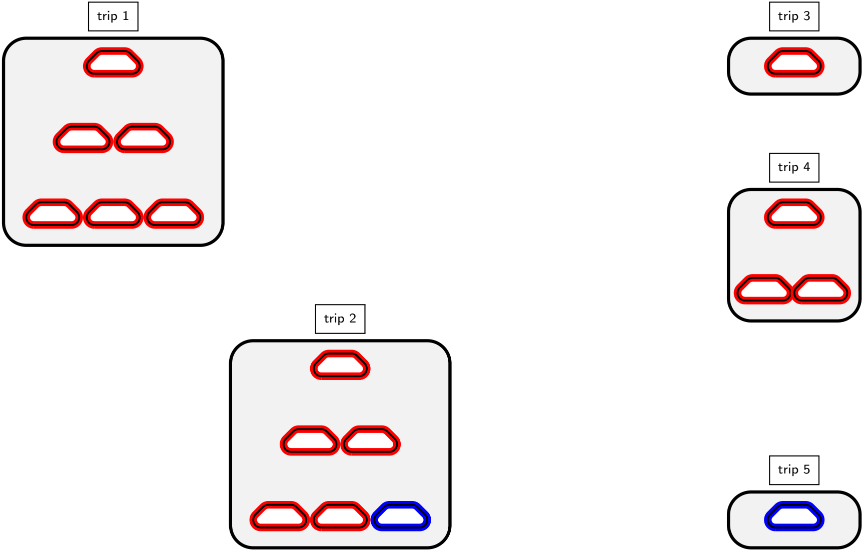
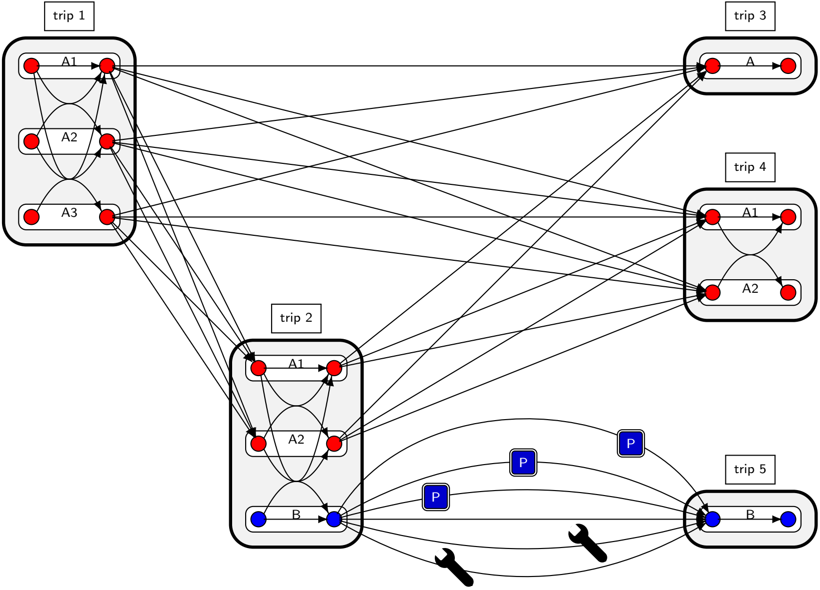
Hypergraph based model



Hypergraph based model



MILP formulation of the RSRP

$$\min \sum_{h \in H} \mathbf{c}_h x_h, \tag{MP}$$

$$\sum_{h \in H(t)} x_h = 1 \qquad \forall t \in T, \tag{1}$$

$$\sum_{h \in H(v)^{\text{in}}} x_h = \sum_{h \in H(v)^{\text{out}}} x_h \qquad \forall v \in V, \tag{2}$$

$$x_h \in \{0, 1\} \qquad \forall h \in H, \tag{3}$$

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MILP formulation of the RSRP

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ROTOR – [Rot]ation [O]ptimizer for [R]ailways

- ROTOR 1.0
  - in production since July 2013
  - integrates all technical details
- ROTOR 2.0
  - in production since July 2014
  - implements re-optimization

ROTOR's Solution Approach

- Solve the root LP of MP by a column generation procedure
- Compute integer solutions from the LP solution via problem specific branching techniques

ROTOR captures

- Operation of trips with coupled vehicles
- Orientation and position requirements for vehicles
- Maintenance requirements for vehicles
- Connection regularity via 'regularity arcs'

Markus Reuther, Ralf Borndoerfer, Thomas Schlechte, and Steffen Weider:
Integrated Optimization of Rolling Stock Rotations for Intercity Railways
accepted for publication in Transportation Science

General idea of template concepts

Template concept for duty scheduling in [Borndörfer et al, 2013]

- Model 'regional' requirements with templates
- Solve pricing problems for templates separately

Regional requirements in *RSRP* sense: Rotations and trips

Borndörfer, Ralf and Langenhan, Andreas and Löbel, Andreas and Schulz, Christof and Weider, Steffen
Duty scheduling templates
Public Transport 5 / 2013

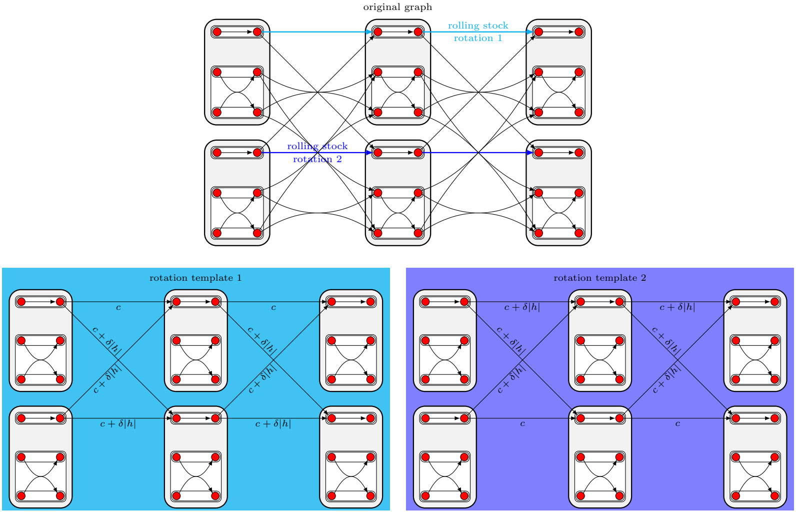
Rotation Template

Definition (Rotation Template)

The hypergraph $G_r = (V_r, A_r, H_r)$ is called *rotation template* for $r \in R$ where:

- G_R originates from G by creating a copy of each node, arc, and hyperarc of G for all $r \in R$
- Deleting all nodes, arcs, and hyperarcs which rotations mismatch r
- Adding a cost penalty of $|h|\delta$ to $\mathbf{c}(h)\ \forall\ h \in H_r$ where h is not part of the reference rotation

Example: Rotation Template



Connection Template

Definition (Connection Template)

Let:

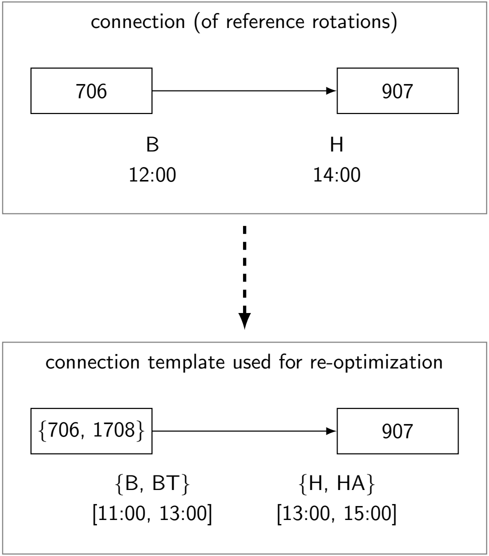
- W set of all connections
- set of similar identifier $I_t \ \forall t \in T$
- set of neighborhood locations $N_l \ \forall l \in L$
- time windows $\Delta_\tau := [\alpha, \beta]$ for each arrival or departure time τ

The *connection template* for reference connection $w \in W$ is the set of hyperarcs $H_w \subseteq \bigcup_{r \in R} H_r$ where for $h \in H_w$ holds:

- trip identifier of h in the sets for similar identifier of w,
- the arrival and departure locations of h are in the corresponding neighborhood sets,
- arrival and departure times fit in the deviation time windows.

We set all connection deviation dependent cost to 0 for $h \in H_w$ if $r(h) = r(w)$.

Example: Connection Template



The Re-Optimization RSRP (Re-Opt RSRP)

Definition (Re-Opt RSRP hypergraph)

Let R be a set of rotations with rotation templates $G_r\ \forall\ r \in R$. The Re-Opt RSRP *hypergraph* is denoted by

$$G_R = \left(\bigcup_{r \in R} V_r, \bigcup_{r \in R} A_r, \bigcup_{r \in R} H_r \right).$$

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MILP formulation of the Re-Opt RSRP

$$\min \sum_{r \in R} \sum_{h_r \in H_r} \mathbf{c}_{h_r} x_{h_r}, \qquad \text{(TMP)}$$

$$\sum_{r \in R} \sum_{h_r \in H_r(t)} x_{h_r} = 1 \qquad \forall t \in T, \qquad (4)$$

$$\sum_{h_r \in H(v_r)^{\text{in}}} x_{h_r} = \sum_{h_r \in H(v_r)^{\text{out}}} x_{h_r} \qquad \forall v_r \in V_r, r \in R \qquad (5)$$

$$x_{h_r} \in \{0, 1\} \qquad \forall h_r \in H, r \in R \qquad (6)$$

Template solution approach

ROTOR's Template Solution Approach

- Solve the root LP of TMP by a column generation procedure
- Solve pricing problems to generate hyperarcs for each template separately
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Instance Characteristics

instance	trips	trip distance(km)	compositions	fleets	disturbed trips	recognizable trips	$\|V\|$	$\|H\|$
RSRP_1	788	555686	2	2	187	22	5702	10026804
RSRP_2	788	555686	2	2	189	22	5702	10027556
RSRP_3	665	265657	2	1	198	184	7680	16465044
RSRP_4	793	430770	11	5	50	7	8350	33113628
RSRP_5	785	426459	11	5	169	120	8330	33069736
RSRP_6	53	37501	5	3	40	40	498	138558
RSRP_7	670	263602	3	2	27	27	7650	16645161
RSRP_8	670	263602	3	2	27	25	7642	16596517

Table : Key numbers of the instances

Results without Rotation Templates

instance	no connection templates: rotation deviations	total costs	rotation dep. costs	computation time(s)	connection templates: rotation deviations	total costs	rotation dep. costs	computation time(s)
RSRP_1	0	2372353	96248	14786	0	2364368	91248	9451
RSRP_2	0	2477205	197853	15653	0	2475174	189853	13396
RSRP_3	200	4146977	301069	5661	200	4145977	300069	4012
RSRP_4	15	5783234	148178	16299	15	5783234	148178	22105
RSRP_5	358	6308657	570314	37543	111	6001832	258314	19878
RSRP_6	0	497744	43323	10	0	476165	20323	51
RSRP_7	462	8565642	3534730	5338	455	8551786	3520730	4508
RSRP_8	484	8613483	3567730	3511	411	8528327	3482730	4091

Table : Computational results with `ROTOR 2.4` and `CPLEX 12.6` without using rotation templates performed on Intel(R) Xeon(R) CPU E31280 3.50 GHz with 16 GB RAM in multi thread mode with four cores.

Results including Rotation Templates

instance	no connection templates: rotation deviations	total costs	rotation dep. costs	computation time(s)	connection templates: rotation deviations	total costs	rotation dep. costs	computation time(s)
RSRP_1	0	2314496	42220	6034	0	2313562	40441	1698
RSRP_2	0	2456665	176853	5265	0	2331487	46166	1775
RSRP_3	0	3946977	101069	2052	0	3866027	28424	1806
RSRP_4	0	5745234	110178	6557	0	5666657	41572	6785
RSRP_5	0	5827728	158314	17172	0	5808472	64592	8650
RSRP_6	0	486744	25323	46	0	459663	4021	34
RSRP_7	7	7959391	2913730	4492	7	6444310	1892170	2306
RSRP_8	6	7983810	2913730	4076	6	6467000	1892290	2087

Table : Key numbers of the computational results of `ROTOR 2.4` and `CPLEX 12.6` with using rotation templates performed on Intel(R) Xeon(R) CPU E31280 3.50 GHz with 16 GB RAM in multi thread mode with four cores.

Conclusion

We have shown that:

- the re-optimization case of the RSRP can be extended by the template concept
- although increasing the graph size templates are a powerful concept that allows us to compute cost minimal rolling stock rotations under a large variety of requirements for re-optimization scenarios
- even in practical size.

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We have shown that:

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Thank you for your attention!

This work has been developed within the Forschungscampus MODAL (Mathematical Optimization and Data Analysis Laboratories) funded by the German Ministry of Education and Research (BMBF).