Comparing two dual relaxations of large scale train timetabling problems

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Train Timetabling Problem

Classical TTP (surveys Lusby et al., 2011; Cacchiani and Toth, 2012)

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- infrastructure network

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Given:

- infrastructure network $G^I = (V^I, A^I)$
  - V^I set of stations, crossings, switches, ...,
  - A^I set of tracks (single and double line tracks),
- set of trains R with
  - predefined routes $G^r = (V^r, A^r) \subseteq G^I$ (paths),
  - starting times $t^r_{\text{start}} \in \mathbb{R}_+$ at first station,
  - running times $\bar{t}^r : A^r \to \mathbb{R}_+$,
- Restrictions:
  - station capacities $c : u \to \mathbb{N}$, $u \in V^I$,
  - headway times $h^a : R \times R \to \mathbb{R}_+$

Goal:

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All train schedules are completely free, no restrictions!

Capacity Restrictions

- at each point in time, at most c_u trains may be at station $u \in V^I$,
- also possible for single directions:

Headway Constraints

- minimal safety distance between two trains $r, r' \in R$ running on the same arc $a = (u, v) \in A^I$,

Headway Constraints

- minimal safety distance between two trains $r, r' \in R$ running on the same arc $a = (u, v) \in A^I$,
- also for ***single line*** tracks and trains running in ***opposite*** directions,

$$\Rightarrow h^a(r, r') \geq \bar{t}_a^r$$

Model

One often used model: ***time-expanded networks*** (*e. g.*, Caprara et al., 2002; Borndörfer and Schlechte, 2007; Fischer et al., 2008)

- discretize time horizon $\rightsquigarrow T = \{1, 2, \dots\}$ (minutes),
- define train graphs $G^r = (V^r, A^r)$, $r \in R$,
- coupling constraints

Model

Train Graphs

$G_T^r = (V_T^r, A_T^r)$ with

$$\begin{aligned} V_T^r =& V^r \times T, \\ A_T^r =& \{((u,t_u),(v,t_v))\colon (u,v) \in A^r, t_v - t_u = \bar{t}^r_{(u,v)}, t_u, t_v \in T\} \\ & \cup \{((u,t_u),(u,t_u+1))\colon u \in V^r_{\text{wait}}, t_u \in T\}, \end{aligned}$$

where V^r_{wait}, $r \in R$, are the stations where r might stop and wait.

Model

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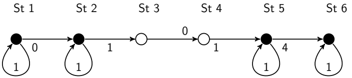
- introduce binary variables $x^r_a \in \{0,1\}$, $r \in R$, $a \in A^r_T$,
- a timetable/schedule of r corresponds to a path

$$P = (u_1, t^r_{\text{start}})\ldots(u_n, t_n) \subseteq G^r_T$$

with

- u_1 ... first station of r,
- u_n ... last station of r,

$\rightsquigarrow \mathcal{P}^r := \{\text{set of feasible train paths in } G^r_T\}$



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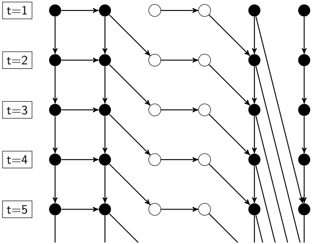
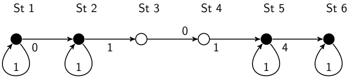
○ train does not stop





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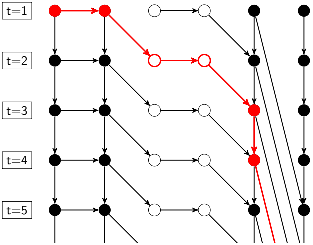


- train schedule corresponds to path

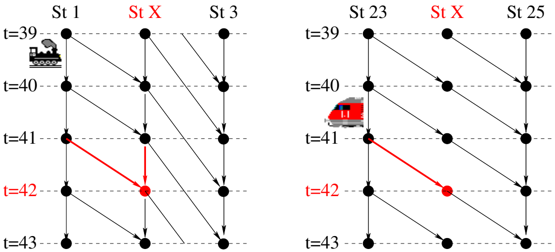
Capacity Constraints

- at most c_u trains are allowed to be at station $u \in V^r$ at the same time:

$$K(u,t) := \{(r,a)\colon a = ((u',t'),(u,t)) \in A^r_T, r \in R\}$$

"arcs corresponding to $r \in R$ being in $u \in V^r$ at $t \in T$"

$$\sum_{(r,a)\in K(u,t)} x^r_a \leq c_u, \qquad u \in V^I, t \in T.$$



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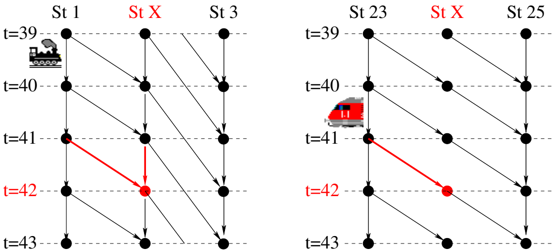

Important, but we ignore them for the rest of the talk!

Headway Constraints

- two train runs
  - $e = ((u, t_u), (v, t_v))$ of $r \in R$ and
  - $e' = ((u, t'_u), (v, t'_v))$ of $r' \in R$

  must not be used both if

$$-h^{(u,v)}(r', r) < t'_u - t_u < h^{(u,v)}(r, r') \qquad (*)$$

$$\rightsquigarrow H := \{\{(r, e), (r', e')\} \colon \text{violate headways eq. } (*)\}$$

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- collect all vectors $x \colon \bigcup_{r \in R} A^r \to \{0, 1\}$ that do not satisfy (*) for all $r, r' \in R$, $a \in A^I$

$$\mathcal{H} := \left\{ x = (x^r)_{r \in R} \colon \forall\, ((r, e), (r', e')) \in H, x_e^r + x_{e'}^{r'} \leq 1 \right\}$$

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- $\mathcal{H}$ is rather complicated, can be described (approximately) in several ways
  - inequality constraints, cutting of infeasible points,
  - model feasible points explicitly

Headway Constraints: Clique Inequalities

use inequalities to describe $\mathcal{H}$

- simplest case:

$$x_e^r + x_{e'}^{r'} \leq 1, \qquad \{(r, e), (r', e')\} \in H,$$

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$$\mathcal{C} := \{\text{subsets of pairwise conflicting train runs}\},$$

then

$$\sum_{(r,e) \in C} x_e^r \leq 1, \qquad C \in \mathcal{C}$$

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- in practise: use approximation

$$\tilde{\mathcal{C}} \subseteq \mathcal{C}, \qquad \rightsquigarrow \qquad \tilde{\mathcal{H}} \supset \mathcal{H}$$

Headway Constraints: Configuration Networks

alternative formulation: configuration networks (Borndörfer and Schlechte, 2007)

- one network $\hat{G}^a = (\hat{V}^a, \hat{A}^a)$ for infrastructure arc $a \in A^I$,

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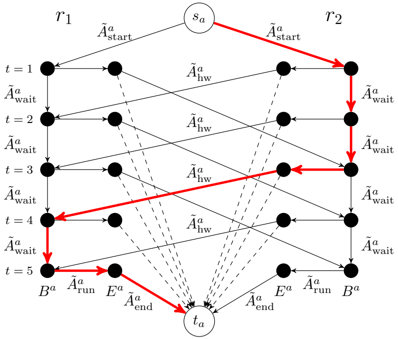
- models ***feasible configuration***, *i. e.*, conflict-free runs of all trains over a,
- for each ***train run arc*** $e = ((u, t_u), (v, t_v))$ one corresponding ***configuration arc*** $e' \in \hat{A}^a$,



- configuration corresponds to s_a-t_a-path in $\hat{G}^a$

$\rightsquigarrow \hat{\mathcal{P}}^a = \{$set of feasible configurations in $\hat{G}^a\}$ relaxation $\rightsquigarrow \tilde{\mathcal{P}}^a_{\text{rlx}} \supseteq \hat{\mathcal{P}}^a$

Models

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- all coupling constraints are ***separated***,
- solved using ***Bundle Methods*** (see, *e.g.*, Hiriart-Urruty and Lemaréchal, 1993)

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  $\rightsquigarrow$ weak bounds,
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Configurations

- coupling constraints easy to separate,
- configuration subproblem hard to solve, but good approximations possible
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Comparing Cliques and Configurations

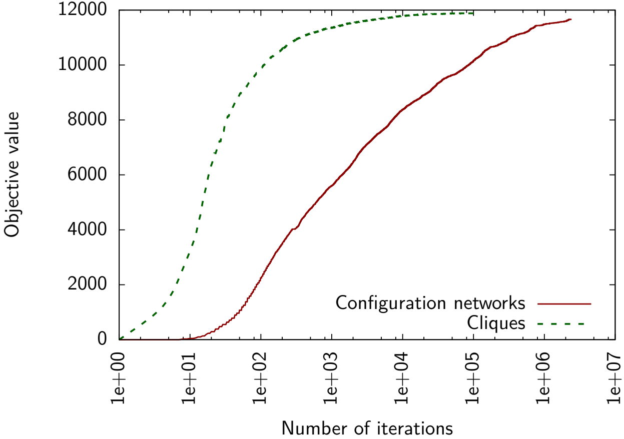
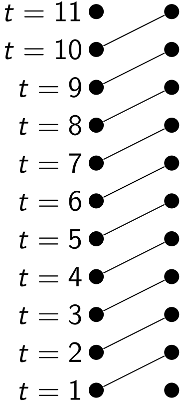


Figure: Objective function after a certain number of iterations for Cliques (dashed line) and Configurations (solid line).

Example: Convergence

- two trains, A more important than B
- headway time: 10 minutes
- optimal: A runs at $t = 1$, B at $t = 11$



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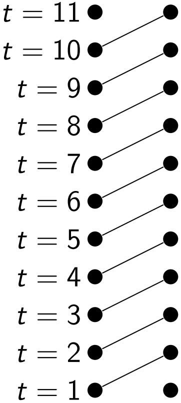

- only the single clique constraint

$$\sum_{t=1}^{10} \left(x_t^A + x_t^B \right) \leq 1$$

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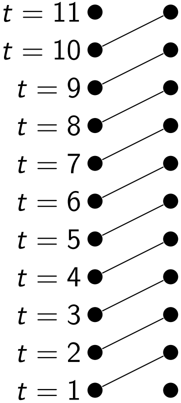

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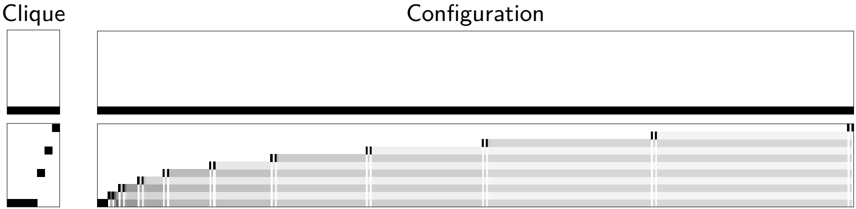
  are required!

Relaxation during the Solution Process



- the single (violated) clique constraint affects ***all*** arcs at the same time,
- the configuration constraints affect ***only a single arc***
  $\rightsquigarrow$ much more iterations are required until all ***Lagrange Multipliers*** are adjusted
- the bundle method does not "see" the structure hidden in the configuration networks

A Combined Approach

Goal: combine good convergence of Cliques with good bounds of Configurations,

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Cliques

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Combined

$$\begin{aligned} \text{maximize} \quad & \sum_{r\in R} \langle w^r, x^r \rangle \\ \text{subject to} \quad & x^r \in \mathcal{P}^r, && r \in R, \\ & Mx = M\tilde{x}, \\ & \tilde{x}^a \in \tilde{\mathcal{P}}^a_{\mathrm{rlx}}, && a \in A^I \end{aligned}$$

- **Clearly:** $\tilde{x}^a \in \hat{\mathcal{P}}^a \Rightarrow b \ge M\tilde{x} = Mx$

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  $\rightsquigarrow$ not directly tractable
- our approach: use a ***scaling bundle method***

Scaling Bundle Method

- the Lagrangian relaxation of the configuration model reads

$$\min_{p} \varphi(y) := \left\{ \sum_{r \in R} \max_{x^r \in \mathcal{P}^r} \langle w^r - p^r, x^r \rangle + \max_{\tilde{x} \in \mathcal{H}} \langle p^r, \tilde{x} \rangle \right\}$$

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- bundle method solves in each iteration the QP

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- it can be shown, that solving the Lagrangian relaxation of the combined model is the same as replacing this subproblem by

$$\min_p \left\{ \hat{\varphi}(p) + \frac{u}{2} \|p - \hat{p}\|^2_{(M^T M)^{-1}} \right\}$$

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$$\min_p \left\{ \hat{\varphi}(p) + \frac{u}{2} \|p - \hat{p}\|^2 \right\}$$

- it can be shown, that solving the Lagrangian relaxation of the combined model is the same as replacing this subproblem by

$$\min_p \left\{ \hat{\varphi}(p) + \frac{u}{2} \|p - \hat{p}\|^2_{(M^T M)^{-1}} \right\}$$

- convergence proof "for free" (*e. g.*, Bonnans et al., 2003)

Scaling Bundle Method

- the Lagrangian relaxation of the configuration model reads

$$\min_p \varphi(y) := \left\{ \sum_{r \in R} \max_{x^r \in \mathcal{P}^r} \langle w^r - p^r, x^r \rangle + \max_{\tilde{x} \in \mathcal{H}} \langle p^r, \tilde{x} \rangle \right\}$$

- bundle method solves in each iteration the QP

$$\min_p \left\{ \hat{\varphi}(p) + \frac{u}{2} \|p - \hat{p}\|^2 \right\}$$

- it can be shown, that solving the Lagrangian relaxation of the combined model is the same as replacing this subproblem by

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- convergence proof “for free” (*e. g.*, Bonnans et al., 2003)
- can also be used with approximations of M

Numerical Experiments

- instances of RAS Problem Solving Competition 2012,
- small network with 100 nodes, 20 trains
- planning horizon of 9 hours,

Numerical Experiments

All three models

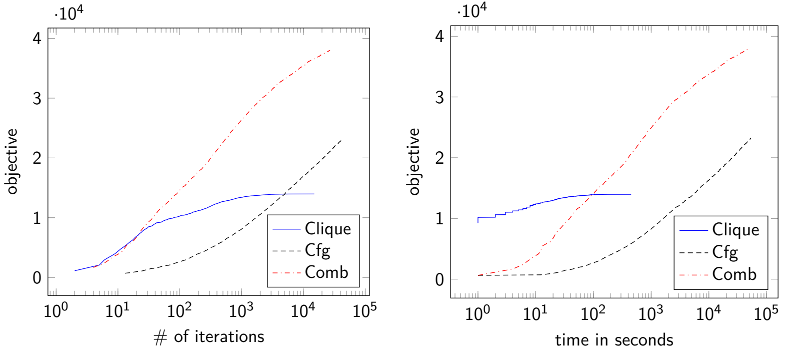

Figure: Objective value after some iterations/time for all three relaxations.

Conclusion

- We compared different (theoretically equivalent) relaxations for the TTP.
- Clique based models converge fast, but have weak bounds.
- Configuration based models converge slowly, but have good bounds.
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Is that all?

- Configuration models are an extended formulation for the TTP.
- allow for formulations of even stronger models (see our ATMOS 2015 paper),
- Combined approach/scaling bundle methods provide the algorithmic tools to solve these models.
- Both approaches together are ongoing work.

Thank you for your attention.
Questions?