TU Clausthal

Modelling Delay Propagation in Railway Networks Using Closed Families of Distributions

F. Kirchhoff, M. Kolonko

Institut für Angewandte Stochastik und OR, TU Clausthal, Germany

Overview

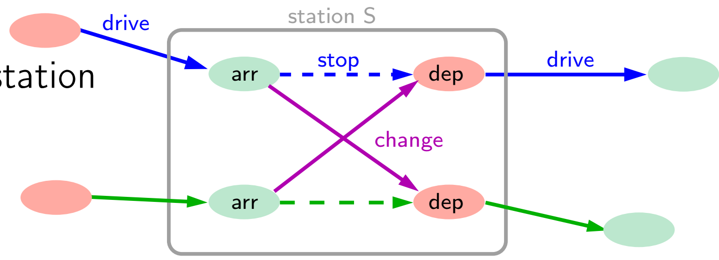
- Periodically served railway network
- Trains are subject to external source delays which are accumulated and propagated.
- **Input**: distribution of source delays

  Output: distribution of propagated delays

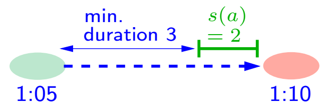
- **Aim**: long-run average performance under typical every day delays, not: severe disturbance or breakdown.
- Can be used to evaluate
  - robustness of timetables,
  - time buffer allocation,
  - waiting time rules
- Fast alternative to Monte-Carlo simulations

Event-Activity-Network

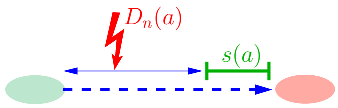
- nodes = events e :
  arrival/departure of a line at station
- arcs = activities a :
  drive, stop, change



- periodic timetable
  - assigns a time to each event e
  - contains a time buffer $s(a) \geq 0$ for each activity

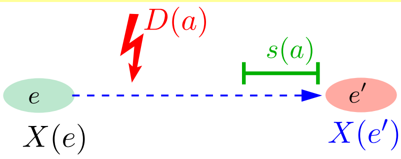


- on each a: random delay $D_n(a)$ during its n-th occurance, $n \in \mathbb{N}$
  $D_1(a), D_2(a), \ldots$ independent, identically distributed

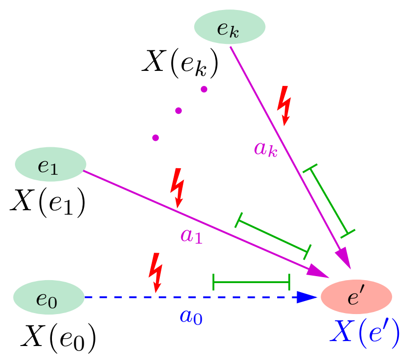


- source delays $D(a)$ occur during activities
  propagated delays $X(e)$ observed at events :
  $X(e)$:= actual time - scheduled time of event e

Delay propagation



- along $a = (e, e')$
  $X(e') = \left[X(e) + D(a) - s(a) \right]^+$

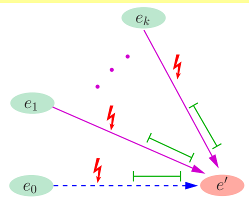


- along change-activity $a_1 = (e_1, e')$

$$X(e') = \max \left\{ \left[X(e_i) + D(a_i) - s(a_i) \right]^+ \mid i = 0, \ldots, k \right\} \quad (\square)$$

- modify for waiting time rule: "wait at most κ minutes for delayed feeders"
- **Input**: cdf F_a of source delays $D(a)$ for each a $\quad F_a(t) = \mathbf{P}(D(a) \leq t)$
  Output: cdf F_e of propagated delays $X(e)$ for all e
- ASSUMPTION **Indep**:
  all delays $X(e_i)$, $D(a_i)$, $i = 0, \ldots, k$, in $(\square)$ are independent

$$X(e') = \max\left\{[X(e_i) + D(a_i) - s(a_i)\,]^+ \mid i = 0,\dots,k\right\} \quad (\Box)$$



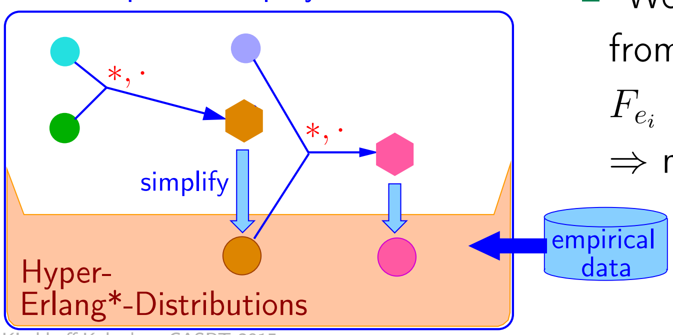
Delay Distributions

- X, Y are independent with cdf F_X, F_Y, then

  $X + Y$ has cdf $F_X * F_Y$ $= \int_0^t F_Y(t-x)\,dF_X(x)$, convolution

  $\max\{X, Y\}$ has cdf $F_X \cdot F_Y$
- cdfs F_{a_i} of $D(a_i)$, cdfs F_{e_i} of $X(e_i)$ } $(\Box)$ $\xrightarrow{*,\,\cdot}$ $F_{e'}$ of $X(e')$
- Needed: family of distributions closed under $*, \cdot$

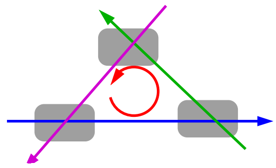
θ-exponential polynomials



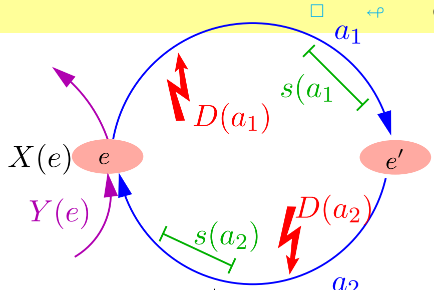
- We can now efficiently determine $F_{e'}$ from F_{a_i} and F_{e_i}

  F_{e_i} must have been determined before

  $\Rightarrow$ network must be cycle-free!



Networks with Cycles



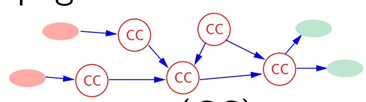
- Iterative approximation of $X(e)$:

  Start with $X^0(\cdot) \equiv 0$ no delays
- $$X^{n+1}(e) = [\quad X^n(e') \quad + D^n(a_2) - s(a_2)]^+ \quad + Y^n(e)$$
  $$= \left[[X^n(e) + D^n(a_1) - s(a_1)]^+ + D^n(a_2) - s(a_2)\right]^+ \quad + Y^n(e)$$
- Distribution of $X^n(e)$ converges against F_e if:

  ★ $\sum_{i \text{ in cycle}} \mathbf{E}\, D(a_i) < \sum_{i \text{ in cycle}} s(a_i)$ buffers compensate for mean delay

  ★ $Y^n(e) \le \kappa < \infty$ bounded propagation from outside

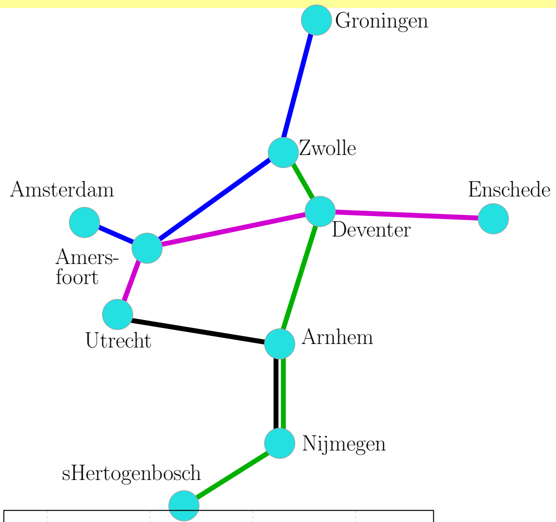
  Analogy to stability in queueing theory

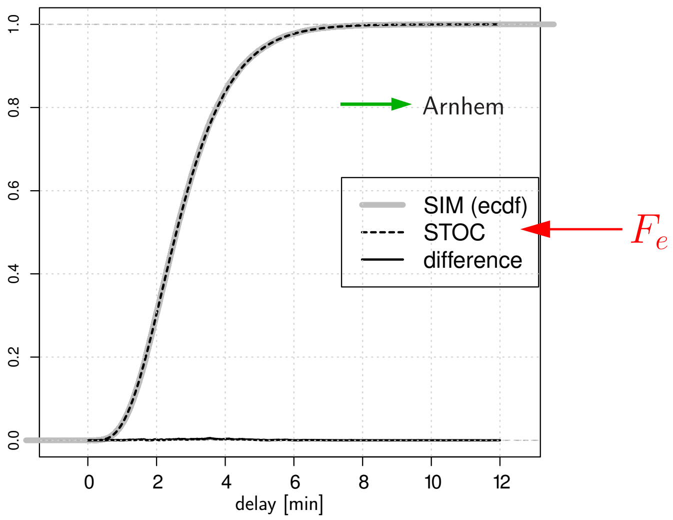


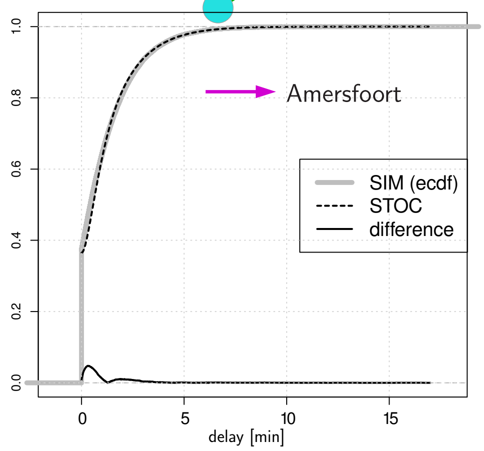
- **Procedure**:
  - partition network into connected components (CC) cycle-free
  - pass through network in topological order
  - for each CC: approximate F_e for all $e \in$ CC as above

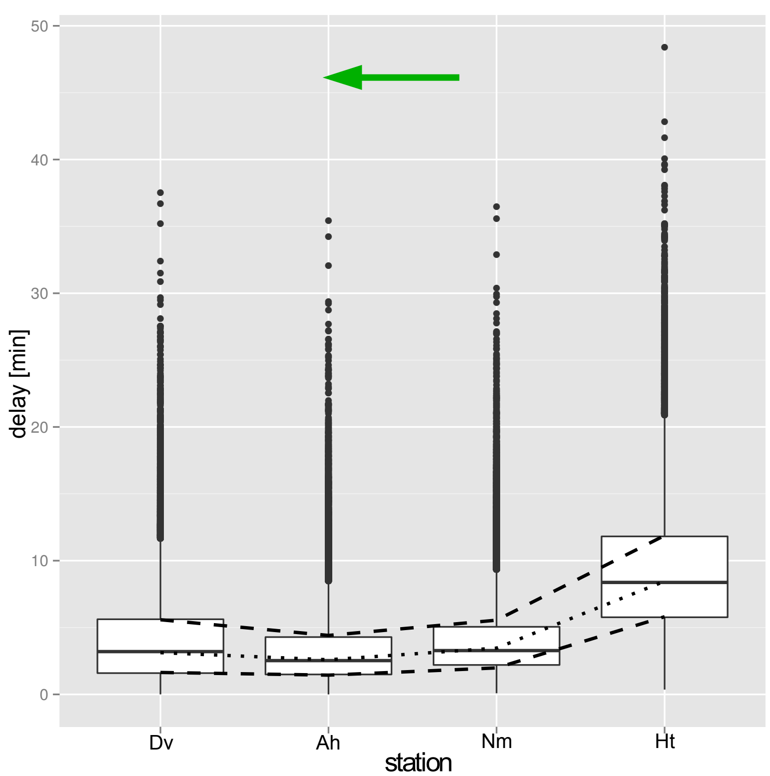
Experimental Results

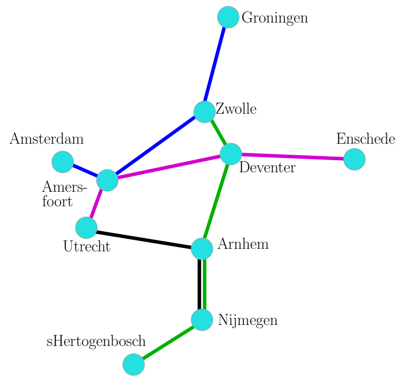
- MEESTER/MUNS 2007
  cycle-free
- source delay distributions M/M 07
- Compare F_e to
  extensive Monte-Carlo-Simulation



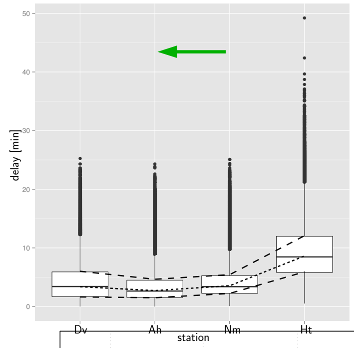


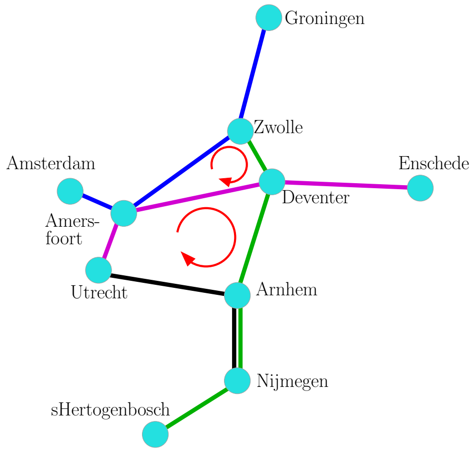


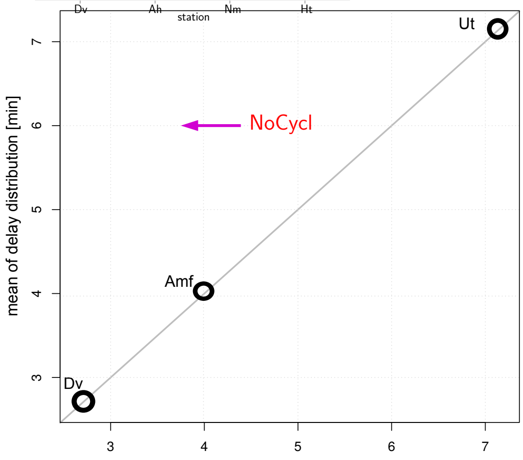


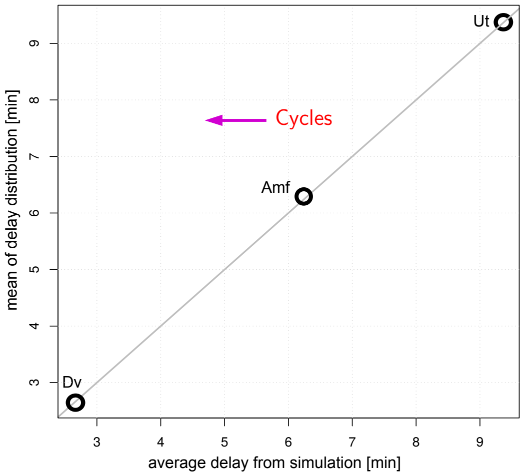


- Network with cycles,
  ★ buffer - delay balance ok, ★ $\kappa = 20$min.









Summary

- We determine the long-run distribution of propagated delays from given source delay distributions,
- in cycles: iterative approximations,
- derived stability conditions ★★,
- results show very good agreement with MC simulation.